Integral Calculus Test #1 Name:_____

Brewer 30 November 2012

Work the following problems on separate paper and staple attached sheets, with problem numbers in order, to this cover sheet. Make sure your name is on each sheet. Read each question carefully. You may use a calculator to check your answers, but **you MUST SHOW ALL WORK in order to receive credit**. Please circle/box your final answers. "Simplified exact answer" means a single, simplified improper fraction (or integer), in terms of π as necessary.

1. Use differentials to approximate the value of the expression. Give an exact answer as well as an approximate answer to five decimal places.

$$\cos\frac{19\pi}{40}$$

2. Find the indefinite integral and check the result by differentiation.

$$\int (\sec x \tan x + 10 \sin x)dx$$

3. Evaluate the definite integral. Give a simplified exact answer.

$$\int_1^2 \frac{x^2 - x + 5}{x^4}dx$$

4. Solve the differential equation.

$$f''(x) = 3x^2 - \sin x\ , \qquad f(0) = 1\ , \qquad f'(0) = 2$$

5. A ball is thrown vertically from a height of 10 feet with initial velocity of 60 feet per second. How high will the ball go? Use $a(t) = -32\frac{ft}{s^2}$ as the acceleration due to gravity. Give a simplified exact answer.

6. Evaluate the sum.

$$\sum_{i=1}^{40}(i^2 + 2i)$$

7. Use the limit process to find the area of the region between the graph of the function and the x-axis over the indicated interval. Sketch the region. Give a simplified exact answer.

$$y = 4x - x^2\ , \qquad [0,2]$$

8. Evaluate the definite integral by the limit definition. Give a simplified exact answer.
Check your answer by computing the definite integral using the Fundamental Theorem of Calculus.

$$\int_1^2 (x^2 + 5)dx$$

9. Sketch the region whose area is given by the definite integral. Then use a geometric formula to evaluate the integral.

$$a)\ \int_{-2}^{4}(x + 4)dx$$

$$b)\ \int_{-4}^{4}\sqrt{16 - x^2}$$

10. Express the limit as a definite integral on the interval $[1,3]$, where c_i is any point in the ith subinterval. Do not solve.

$$a)\ \lim_{\|x\|\to 0}\sum_{i=1}^{n}(5{c_i}^2 + 2)\Delta x_i$$

$$b)\ \lim_{\|x\|\to 0}\sum_{i=1}^{n}{c_i}^3(5{c_i}^2 + 5)\Delta x_i$$

Bonus:

A. Find the function whose tangent has slope $5x^4 - x + 5$ for each value of x, and whose graph passes through the point $(0,8)$.

B. Use the limit process to find the area of the region bounded by

$$x = 5y - y^2\ ,\ x = 0\ ,\ y = 2\ ,\ y = 5$$