Special Factoring

$a^2 - b^2 = (a-b)(a+b)$
$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$
$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Reciprocal Identities

$\csc x = \frac{1}{\sin x}$, $\sin x = \frac{1}{\csc x}$, $\sec x = \frac{1}{\cos x}$
$\cos x = \frac{1}{\sec x}$, $\cot x = \frac{1}{\tan x}$, $\tan x = \frac{1}{\cot x}$

Ratio Identities

$\tan x = \frac{\sin x}{\cos x}$, $\cot x = \frac{\cos x}{\sin x}$

Odd-Even Identities

$\cos(-x) = \cos x$, $\sin(-x) = -\sin x$, $\tan(-x) = -\tan x$
$\sec(-x) = \sec x$, $\csc(-x) = -\csc x$, $\cot(-x) = -\cot x$

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$, $1 + \cot^2 x = \csc^2 x$, $\tan^2 x + 1 = \sec^2 x$

Sum and Difference Identities

$\sin(a+b) = \sin a \cos b + \cos a \sin b$
$\sin(a-b) = \sin a \cos b - \cos a \sin b$
$\cos(a+b) = \cos a \cos b - \sin a \sin b$
$\cos(a-b) = \cos a \cos b + \sin a \sin b$
$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$, $\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$

Cofunction Identities

$\sin\left(\frac{\pi}{2} - x\right) = \cos x$, $\cos\left(\frac{\pi}{2} - x\right) = \sin x$
$\tan\left(\frac{\pi}{2} - x\right) = \cot x$, $\cot\left(\frac{\pi}{2} - x\right) = \tan x$
$\csc\left(\frac{\pi}{2} - x\right) = \sec x$, $\sec\left(\frac{\pi}{2} - x\right) = \csc x$

Double-Angle Identities

$\sin 2x = 2 \sin x \cos x$
$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$
$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$

Half-Angle Identities

$\sin \frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}}$, $\cos \frac{x}{2} = \pm\sqrt{\frac{1 + \cos x}{2}}$

$$\tan \frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}$$

Binomials

$n! = n(n-1)(n-2)(n-3)\cdots 3 \cdot 2 \cdot 1$

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$ = the number of ways to choose k objects out of a set containing n objects

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

$\textit{The } (k+1)^{st} \textit{ term of } (a+b)^n \textit{ is } \binom{n}{k} a^{n-k} b^k$

Properties of Exponential Functions:

$a^m a^n = a^{m+n}$
$\frac{a^m}{a^n} = a^{m-n}$
$(a^m)^n = a^{mn}$

Properties of Logs:

Product Rule: $\log_a MN = \log_a M + \log_a N$
Power Rule: $\log_a M^P = p \log_a M$
Quotient Rule: $\log_a \frac{M}{N} = \log_a M - \log_a N$
Change of Base Formula: $\log_b M = \frac{\log_a M}{\log_a b}$
Other Properties:
$\log_a a = 1$ $\quad \log_a 1 = 0$ $\quad \log_a a^x = x$ $\quad a^{\log_a x} = x$

Arithmetic Sequences/Series:

Definition: A sequence is arithmetic if there exists a number d, called the **common difference**, such that $\boldsymbol{a_{n+1} = a_n + d}$ for any integer $n \geq 1$.

The **nth term** of an arithmetic sequence is given by
$\boldsymbol{a_n = a_1 + (n-1)d}$, for any integer $n \geq 1$.

The **sum of the first n terms** of an arithmetic sequence is given by
$\boldsymbol{S_n = \frac{n}{2}(a_1 + a_n)}$

Geometric Sequences/Series:

Definition: A sequence is geometric is there is a number r, called the **common ratio**, such that
$\boldsymbol{\frac{a_{n+1}}{a_n} = r}$, or $a_{n+1} = a_n r$, for any integer $n \geq 1$.

The **nth term of a geometric sequence** is given by
$\boldsymbol{a_n = a_1 r^{n-1}}$, for any integer $n \geq 1$.

The **sum of the first n terms** of a geometric sequence is given by
$\boldsymbol{S_n = \frac{a_1(1-r^n)}{1-r}}$, for any $r \neq 1$.

When $|r| < 1$, the **limit or sum of an infinite geometric series** is given by
$\boldsymbol{S_\infty = \frac{a_1}{1-r}}$

Limit Rules

Basic Limits

Let $b, c \in \mathbb{R}$, $n > 0$ *an integer*, $f, g -$ *functions*,

$\lim_{x \to c} f(x) = L$, $\lim_{x \to c} g(x) = K$

1. Constant $\lim_{x \to c} b = b$
2. Identity $\lim_{x \to c} x = c$
3. Polynomial $\lim_{x \to c} x^n = c^n$
4. Scalar Multiple $\lim_{x \to c} [bf(x)] = bL$
5. Sum or Difference $\lim_{x \to c} [f(x) \pm g(x)] = L \pm K$
6. Product $\lim_{x \to c} [f(x)g(x)] = LK$
7. Quotient $\lim_{x \to c} \left[\frac{f(x)}{g(x)}\right] = \frac{L}{K}$, $K \neq 0$
8. Power $\lim_{x \to c} [f(x)]^n = L^n$

Note: If substitution yields $\frac{0}{0}$, an indeterminate form, the expression must be rewritten in order to evaluate the limit.

Continuity at a point

A function f is continuous at c if the following 3 conditions are met:

1. $f(c)$ is defined
2. Limit of $f(x)$ exists when x approaches c
3. Limit of $f(x)$ when x approaches c is equal to $f(c)$

$$\lim_{x \to c} f(x) = f(c)$$

$\varepsilon - \delta$ Definition of the Limit:

$\lim_{x \to c} f(x) = L$ if given $\varepsilon > 0$, there exists a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

The Squeeze Theorem:

If $f(x) \leq g(x) \leq h(x)$ *and* $\lim_{x \to c} f(x) = L = \lim_{x \to c} h(x)$, then $\lim_{x \to c} g(x) = L$.

Special Limits Derived by Squeeze Theorem:

$$\lim_{x \to 0} \frac{\sin x}{x} = 1 \quad ; \quad \lim_{x \to 0} \frac{1 - \cos x}{x} = 0$$

Derivative Rules

Power Rule:

$$\frac{d}{dx}[x^n] = nx^{n-1}$$

Constant Multiple Rule:

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$$

Sum & Difference:

$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

Chain Rule:

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

Product Rule:

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

Quotient Rule:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$$

Exponential and Logarithmic Functions:

$$\frac{d}{dx}[a^x] = a^x \ln a$$
$$\frac{d}{dx}[\log_a x] = \frac{1}{x \ln a}$$
$$\frac{d}{dx}[e^x] = e^x$$
$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$

Trig Functions:

$$\frac{d}{dx}[\sin x] = \cos x$$
$$\frac{d}{dx}[\cos x] = -\sin x$$
$$\frac{d}{dx}[\tan x] = \sec^2 x$$
$$\frac{d}{dx}[\cot x] = -\csc^2 x$$
$$\frac{d}{dx}[\sec x] = \sec x \tan x$$
$$\frac{d}{dx}[\csc x] = -\csc x \cot x$$

Inverse Trig Functions:

$$\frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1 - x^2}}$$
$$\frac{d}{dx}[\arctan x] = \frac{1}{1 + x^2}$$
$$\frac{d}{dx}[\text{arcsec}\, x] = \frac{1}{|x|\sqrt{x^2 - 1}}$$
$$\frac{d}{dx}[\arccos x] = \frac{-1}{\sqrt{1 - x^2}}$$
$$\frac{d}{dx}[\text{arccot}\, x] = \frac{-1}{1 + x^2}$$
$$\frac{d}{dx}[\text{arccsc}\, x] = \frac{-1}{|x|\sqrt{x^2 - 1}}$$