

Special Factoring

$$a^2 - b^2 = (a - b)(a + b)$$

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Reciprocal Identities

$$\csc x = \frac{1}{\sin x}, \quad \sin x = \frac{1}{\csc x}, \quad \sec x = \frac{1}{\cos x}$$

$$\cos x = \frac{1}{\sec x}, \quad \cot x = \frac{1}{\tan x}, \quad \tan x = \frac{1}{\cot x}$$

Ratio Identities

$$\tan x = \frac{\sin x}{\cos x}, \quad \cot x = \frac{\cos x}{\sin x}$$

Odd-Even Identities

$$\cos(-x) = \cos x, \quad \sin(-x) = -\sin x, \quad \tan(-x) = -\tan x$$

$$\sec(-x) = \sec x, \quad \csc(-x) = -\csc x, \quad \cot(-x) = -\cot x$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1, \quad 1 + \cot^2 x = \csc^2 x, \quad \tan^2 x + 1 = \sec^2 x$$

Sum and Difference Identities

$$\sin(a + b) = \sin a \cos b + \cos a \sin b$$

$$\sin(a - b) = \sin a \cos b - \cos a \sin b$$

$$\cos(a + b) = \cos a \cos b - \sin a \sin b$$

$$\cos(a - b) = \cos a \cos b + \sin a \sin b$$

$$\tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}, \quad \tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x, \quad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x, \quad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

$$\csc\left(\frac{\pi}{2} - x\right) = \sec x, \quad \sec\left(\frac{\pi}{2} - x\right) = \csc x$$

Double-Angle Identities

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

Half-Angle Identities

$$\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}, \quad \cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$\tan \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}$$

Binomials

$$n! = n(n - 1)(n - 2)(n - 3) \cdots 3 \cdot 2 \cdot 1$$

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$ = the number of ways to choose k objects out of a set containing n objects

$$(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$$

The $(k + 1)^{\text{st}}$ term of $(a + b)^n$ is $\binom{n}{k} a^{n-k} b^k$

Properties of Exponential Functions:

$$a^m a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

Properties of Logs:

$$\text{Product Rule: } \log_a MN = \log_a M + \log_a N$$

$$\text{Power Rule: } \log_a M^p = p \log_a M$$

$$\text{Quotient Rule: } \log_a \frac{M}{N} = \log_a M - \log_a N$$

$$\text{Change of Base Formula: } \log_b M = \frac{\log_a M}{\log_a b}$$

Other Properties:

$$\log_a a = 1 \quad \log_a 1 = 0 \quad \log_a a^x = x \quad a^{\log_a x} = x$$

Arithmetic Sequences/Series:

Definition: A sequence is arithmetic if there exists a number d , called the **common difference**, such that $a_{n+1} = a_n + d$ for any integer $n \geq 1$.

The **n^{th} term** of an arithmetic sequence is given by

$$a_n = a_1 + (n - 1)d, \text{ for any integer } n \geq 1.$$

The **sum of the first n terms** of an arithmetic sequence is given by

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Geometric Sequences/Series:

Definition: A sequence is geometric if there is a number r , called the **common ratio**, such that

$$\frac{a_{n+1}}{a_n} = r, \text{ or } a_{n+1} = a_n r, \text{ for any integer } n \geq 1.$$

The **n^{th} term of a geometric sequence** is given by

$$a_n = a_1 r^{n-1}, \text{ for any integer } n \geq 1.$$

The **sum of the first n terms** of a geometric sequence is given by

$$S_n = \frac{a_1(1-r^n)}{1-r}, \text{ for any } r \neq 1.$$

When $|r| < 1$, the **limit or sum of an infinite geometric series** is given by

$$S_\infty = \frac{a_1}{1-r}$$

Derivative Rules

Power Rule:

$$\frac{d}{dx}[x^n] = nx^{n-1}$$

Constant Multiple Rule:

$$\frac{d}{dx}[cf(x)] = c \frac{d}{dx}[f(x)]$$

Sum & Difference:

$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

Product Rule:

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

Quotient Rule:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$$

Chain Rule:

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

Trig Functions:

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

$$\frac{d}{dx}[\tan x] = \sec^2 x$$

$$\frac{d}{dx}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}[\sec x] = \sec x \tan x$$

$$\frac{d}{dx}[\csc x] = -\csc x \cot x$$

Exponential and Logarithmic Functions:

$$\frac{d}{dx}[a^x] = a^x \ln a$$

$$\frac{d}{dx}[\log_a x] = \frac{1}{x \ln a}$$

$$\frac{d}{dx}[e^x] = e^x$$

$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$

Inverse Trig Functions:

$$\frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\arctan x] = \frac{1}{1+x^2}$$

$$\frac{d}{dx}[\operatorname{arcsec} x] = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}[\arccos x] = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\operatorname{arccot} x] = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}[\operatorname{arccsc} x] = \frac{-1}{|x|\sqrt{x^2-1}}$$