Differential Calculus | Practice Problems for Final Exam
Brewer | Fall 2010

1. Use the product rule to differentiate the function.
a. $f(x) = (x^2 + \sin x)(3x - 15)$
b. $f(x) = \sqrt{x}\tan x$

2. Use the quotient rule to differentiate the function.
a. $f(x) = \frac{\cos x}{x^2}$
b. $f(x) = \frac{2(\cos x - 1)}{3\sin x}$

3. Find $f'(3)$ given that $g(3) = 2,\ g'(3) = -1,\ h(3) = 5,$ and $h'(3) = -2$
a. $f(x) = 2g(x)h(x)$
b. $f(x) = 2g(x) + h(x)$
c. $f(x) = 7 + \frac{g(x)}{h(x)}$

4. Find the second derivative of the function.
a. $f(x) = \frac{\tan x - 2}{3}$
b. $f(x) = \csc^2 \pi x$

5. Find the derivative of the function.
a. $f(x) = \sqrt{\frac{x}{x^3 - 2x}}$
b. $f(x) = \left(\frac{\sin x}{x^3 - 2x}\right)^3$
c. $f(x) = 3x - 5\cot(\pi x)^2$
d. $f(x) = \ln(\tan^{-1}(2x))$
e. $f(x) = 5^{\csc x}\sqrt{x^3 - 7x}$

6. Find an equation of the tangent line to the graph of f at the indicated point.
$f(x) = \sqrt{2x^2 - 7}\,,\ (2, 1)$

7. The length of a rectangle is given by $2(t^2 + 1)$ and its height is $\sqrt{t + 5}$ where t is time in seconds and dimensions are in centimeters.
a. Find the average rate of change of the area from time 4 to time 11.
b. Find the instantaneous rate of change of the area at time 4.

8. Find $\frac{dy}{dx}$ by implicit differentiation and evaluate the derivative at the indicated point.
a. $x^{2/3} + y^{2/3} = 5,\ (8, 1)$
b. $x\cos y = 1\,,\ \left(2, \frac{\pi}{3}\right)$

9. Find $\frac{d^2y}{dx^2}$ in terms of x and y.
a. $x^2 = y^3$
b. $1 - xy = x - y$

10. All edges of a cube are expanding at a rate of 3 centimeters per second. How fast is the volume changing when each edge is 10 centimeters?

11. A conical tank (with vertex down) is 6 feet across and 12 feet deep. If water is flowing into the tank at a rate of 10 cubic feet per minute, find the rate of change of the depth of the water when the water is 8 feet deep.

12. An airplane is flying in still air with an airspeed of 240 miles per hour. If it is climbing at an angle of 30°, find the rate at which it is gaining altitude.

13. Find the limit.

$a.\ \lim\limits_{x\to 0} \frac{\sin x\ (1-\cos x)}{2x^2}$

$b.\ \lim\limits_{x\to 3^-} f(x), where\ f(x) = \begin{cases} \frac{x+2}{2}, x \leq 3 \\ \frac{12-2x}{3}, x > 3 \end{cases}$

$c.\ \lim\limits_{x\to 3^+} \frac{x^2}{x^2-9}$

$d.\ \lim\limits_{x\to\infty} \frac{3x^3+2}{9x^3-2x^2+7}$

$e.\ \lim\limits_{x\to -\infty} \frac{3x-5}{\sqrt{4x^2+2x-1}}$

$f.\ \lim\limits_{x\to 1} \frac{\ln x^2}{x^2-1}$

$g.\ \lim\limits_{x\to\infty} \frac{e^{x/2}}{x^3}$

$h.\ \lim\limits_{x\to\infty} \frac{\sin x}{x}$

$i.\ \lim\limits_{x\to\infty} (1+x)^{1/x}$

14. Find the derivative of the function using the definition (limit of the difference quotient)
$f(x) = x^3 - 12x$

15. A rectangle is bounded by the x-axis and the semi-circle $y = \sqrt{25-x^2}$ (see figure on p. 217 of textbook). What length and width should the rectangle have so that its area is a maximum?

16. A solid is formed by adjoining two hemispheres to the ends of a right circular cylinder. The total volume of the solid is 12 cubic centimeters. Find the radius of the cylinder that produces the minimum surface area.

17. Locate the absolute extrema on the closed interval.
$f(x) = x^3 - 12x, \qquad [0,4]$

18. Determine whether the Mean Value Theorem can be applied to f on the closed interval, and if so, find all values of c such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

$f(x) = \frac{x+1}{x}, \left[\frac{1}{2}, 2\right]$

19. Find the critical numbers of f (if any), find the open intervals on which the function is increasing or decreasing, and locate all relative extrema.

$f(x) = \frac{x^2-2x+1}{x+1}$

20. Find the points of inflection and discuss the concavity of the graph of the function.
$f(x) = 2x^4 - 8x + 3$