Differential Calculus Test #3 Name:_____
Brewer 12 May 2010

Read each question carefully. Work questions on separate paper and staple to this sheet. You do not have to simplify answers that are in terms of x and/or y (as long as all derivatives are taken). Numerical answers (i.e. derivatives evaluated at a point and specific values for rates) SHOULD be simplified and given as EXACT values (i.e. fractions NOT decimals). If any instructions are unclear, please ask. Circle your final answers.

1. A jumbo waffle cone from Sarah's Tasty Ice Cream Shoppe is 10 inches tall and has a 4 inch diameter at the top of the cone. Yesterday, my cone had a leak! Instead of eating it super fast, I decided to compare the rate of change of volume of ice cream to the rate of change of height of ice cream in the cone. How fast is the ice cream leaking out (in cubic inches per minute) when there are 5 inches of ice cream in the cone, if the height of ice cream in the cone is changing at a rate of 1 inch every 5 minutes?

2. $x^3 + y^2 = 10$
a. Find y' in terms of x and y.
b. Find y'' in terms of x and y.

3. $f(x) = (x-3)^2(x+2)$
a. Find the critical numbers of the function.
b. Find the absolute maximum and absolute minimum (if any) on the closed interval $[-1,1]$.
c. Find all open intervals on the function's domain on which it is increasing or decreasing.

4. $f(x) = (x+3)^2(x-1)$
a. Determine if Rolle's Theorem can be applied on the closed interval $[-3,1]$, and if so, find all values of c in $(-3,1)$ guaranteed by that theorem such that $f'(c) = 0$.
b. Determine if the Mean Value Theorem can be applied on the closed interval $[-2,2]$, and if so, find all values of c in $(-2,2)$ guaranteed by that theorem such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

Bonus: Find the polynomial $P_2(x) = a_0 + a_1x + a_2x^2$ whose value and first two derivatives agree with the value and first two derivatives of $f(x) = \cos x$ at the point $x = 0$. This polynomial is called the second-degree Taylor polynomial of $f(x) = \cos x$ at $x = 0$.