Integral Calculus - Final Exam Study Guide

1. Use differentials to approximate the value of the expression. Give an exact answer.

$f(c + \Delta x) \approx f(c) + f'(c)\Delta x$

2. Position/Velocity/Acceleration Problem (not necessarily free-fall)

$s(t) = position\ at\ time\ t$
$v(t) = s'(t) = velocity\ at\ time\ t$
$a(t) = v'(t) = s''(t) = acceleration\ at\ time\ t$

3. Use the limit process to find the area of the region between the graph of the function and the x-axis over the indicated interval. Sketch the region. Give an exact answer. Check your answer by setting up the definite integral and using the 1st Fundamental Theorem of Calculus.

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(a + \frac{b-a}{n} i\right) \cdot \frac{b-a}{n}$$

4. Find the value(s) of c guaranteed by the Mean Value Theorem for Integrals for the function over the indicated interval.

If $f(x)$ is continuous on $[a, b]$, then there exists $c \in [a, b]$ such that $f(x) = \frac{1}{b-a}\int_a^b f(t)dt$

5. Area of a region bounded by the graphs of multiple functions.

6. Volume of a solid of revolution

7. Find a particular solution to the differential equation based on the given initial conditions.

8. Find F'(x), given $F(x) = \int_a^{g(x)} f(t)dt$ (2nd fundamental theorem of calculus)

9. Evaluate the indefinite integrals.

10. Evaluate the definite integrals.

Integration by parts, u-substitution, trigonometric substitution, partial fractions, long division, logs, inverse trig functions, Pythagorean and double-angle trig identities, graphically using geometric formulas (triangles, circles, rectangles)

Given formulas:

$$\int \tan u \, du = -\ln|\cos u| + c$$
$$\int \cot u \, du = \ln|\sin u| + c$$
$$\int \sec u \, du = \ln|\sec u + \tan u| + c$$
$$\int \csc u \, du = -\ln|\csc u + \cot u| + c$$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin\frac{u}{a} + c$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\arctan\frac{u}{a} + c$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + c$$

$$\text{Arc length} = \int_a^b \sqrt{1 + [f'(x)]^2} \, dx$$

$$\text{Volume} = \int_a^b \pi r^2 dx$$

$$\text{Surface area} = \int_a^b 2\pi r\sqrt{1 + [f'(x)]^2} \, dx$$

$$\sum_{i=1}^{n} c = cn$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(a + \frac{b-a}{n} i\right) \cdot \frac{b-a}{n}$$

If $f(x)$ is continuous on $[a, b]$, then there exists $c \in [a, b]$ such that $f(c) = \frac{1}{b-a}\int_a^b f(x)dx$

$f(c + \Delta x) \approx f(c) + f'(c)\Delta x$

$s(t) = \textit{position at time } t$
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$\cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$