Integral Calculus – Practice Problems for Test #1

Find the differential dy.

1. $y = x(1 - \cos x)$
2. $y = \sqrt{36 - x^2}$

3. Use differentials to approximate the value of the expression.
   $\sqrt[3]{63}$

Find the indefinite integral and check the result by differentiation.

4. $\int \frac{x^2-3x+4}{x^4}\,dx$
5. $\int \left(\frac{4}{x^4} + \sin x\right) dx$
6. $\int (5\cos x - 2\sec^2 x)\,dx$

7. Solve the differential equation.
   $f'(x) = 6x^2 - \cos x\ , \qquad f(0) = 1$

8. Find the particular solution of the differential equation $f'(x) = -2x$ whose graph passes through the point $(-1,1)$.

9. Find the particular solution of the differential equation $f''(x) = 6(x-1)$ whose graph passes through the point $(2,1)$ and is tangent to the line $3x - y - 5 = 0$ at that point.

10. A ball is thrown vertically from a height of 5 feet with initial velocity of 50 feet per second. How high will the ball go? (Use $a(t) = -32$ feet per second per second as the acceleration due to gravity. Neglect air resistance.)

11. The rate of growth dP/dt of a population of bacteria is proportional to the cube root of t, where P is the population size and t is the time in days $(0 \le t \le 10)$. That is, $\frac{dP}{dt} = k\sqrt[3]{t}$. The initial size of the population is 1000. After 1 day the population has grown to 1100. Estimate the population after 10 days.

12. A particle, initially at rest, moves along the x-axis such that its acceleration at time $t > 0$ is given by $a(t) = sin\, t$. At time $t = 0$, its position is $x = 5$.
    (a) Find the velocity and position function for the particle.
    (b) Find the values of t for which the particle is at rest.

Evaluate the sum.

13. $\sum_{i=1}^{20}(3i^2 + 2i + 4)$
14. $\sum_{i=1}^{40} i(i^2 + 1)$

Find the limit.

15. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n^3}(i-2)^2$

16. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3i}{n^2}$

Use the limit process to find the area of the region between the graph of the function and the x-axis over the indicated interval. Sketch the region.

17. $y = 2x - x^3$, $[0,1]$
18. $y = x^2 - x^3$, $[-1,0]$

Use the limit process to find the area of the region between the graph of the function and the y-axis over the indicated interval. Sketch the region.

19. $f(y) = y^2$, $0 \le y \le 3$

20. Evaluate the definite integral by the limit definition.

$$\int_1^3 (2x + 5)dx$$

Sketch the region whose area is given by the definite integral. Then use a geometric formula to evaluate the integral.

21. $\int_{-2}^{2}(x + 3)dx$
22. $\int_{-3}^{3}(3 - |x|)dx$
23. $\int_{-2}^{2} \sqrt{4 - x^2}dx$

Express the limit as a definite integral on the interval [2,3], where c_i is any point in the ith subinterval.

24. $\lim_{\|\Delta\|\to 0} \sum_{i=1}^{n}({c_i}^2 + 5)\, \Delta x_i$
25. $\lim_{\|\Delta\|\to 0} \sum_{i=1}^{n} c_i(2{c_i}^2 + 1)\, \Delta x_i$

Given $\int_0^3 f(x) = 5$ and $\int_3^7 f(x) = 11$, find

26. $\int_3^0 4f(x)dx$
27. $\int_3^3 10f(x)dx$
28. $\int_7^0 f(x)dx$
29. $\int_0^7 4f(x)dx$
30. $\int_7^3 -f(x)dx$

Given $\int_1^5 f(x) = 5$ and $\int_1^5 g(x) = 14$, find

31. $\int_1^5 [4f(x) - g(x)]\, dx$
32. $\int_1^5 [5f(x) + g(x)]\, dx$