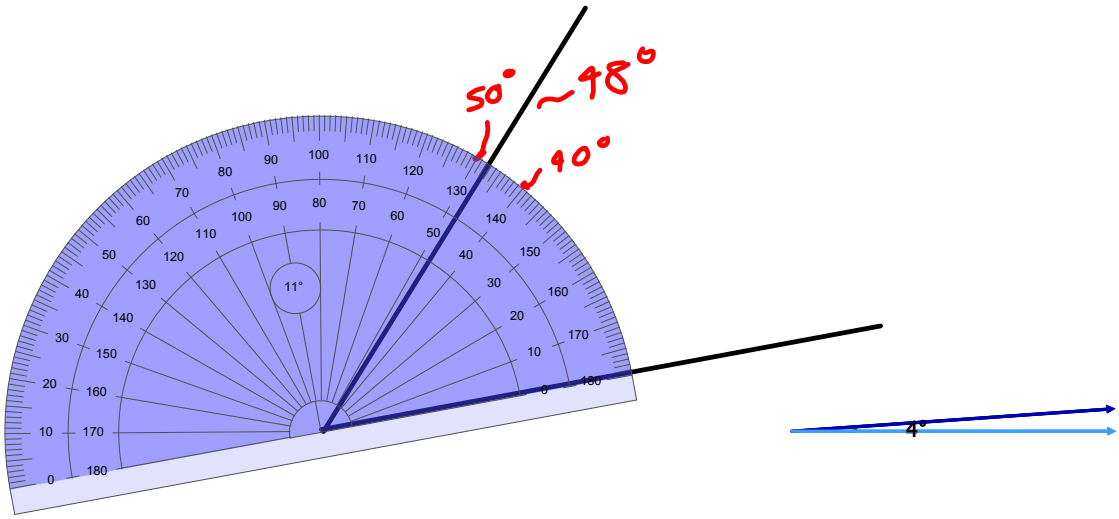


2.1 – Conditional Statements

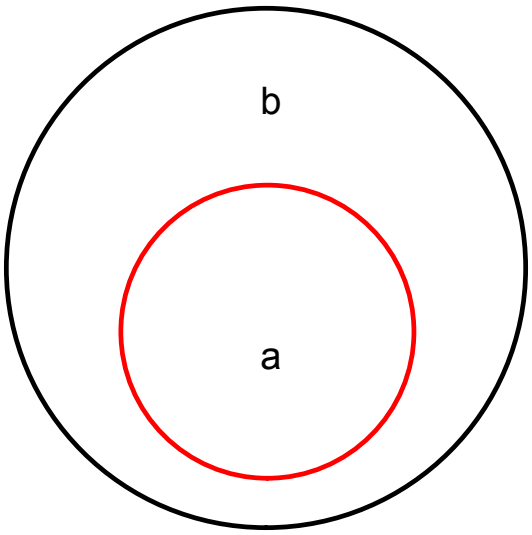
Conditional Statement: "If a, then b." or "a implies b."

a = hypothesis

b = conclusion

$a \rightarrow b$

Euler diagram



2.1 #7-11

"If it is snowing, then it is cold outside." true

"If it is cold outside, then it is snowing." false

7. Do both statements have the same hypothesis? no conclusions

8. Are they both true? Why or why not?

no

9. Do they mean the same thing? no

10. Rewrite the first statement so that the conclusion is stated before the hypothesis. It is cold outside if it is snowing.

11. Is the statement that you wrote true?

yes

Rewrite the sentences in "if-then" form.

16. "Smokey Bear wouldn't have to do commercials for a living if money grew on trees."

If money grew on trees, then Smokey Bear would not have to do commercials.

17. "All architects use geometry."

If a person is an architect, then that person uses geometry.

19. "Use the stairs instead of the elevator in case of fire."

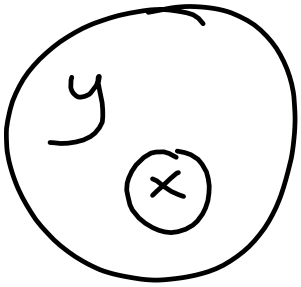
If there is a fire, then use the stairs instead of the elevator.

20. "No vampire casts a shadow."

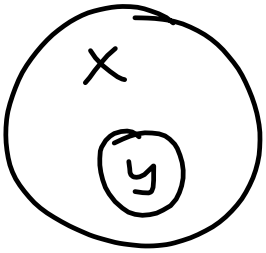
If a person is a vampire, then they will not cast a shadow.

27. Draw Euler diagrams to represent the statements.

"If x, then y."



"If y, then x."



29. Which diagram also illustrates the statement "If not y, then not x"?

If x, then y — contrapositive

2.2 – Definitions

When we define a word in mathematics, the word and its definition are understood to have the exact same meaning.

For example, if we define a "triangle" as "a polygon that has three sides," then both of the following statements are true:

"If a figure is a triangle, then it is a polygon that has three sides."

"If a figure is a polygon that has three sides, then it is a triangle."

For statement "If a, then b." $a \rightarrow b$

Its converse is "If b, then a." $b \rightarrow a$

In general, if a statement is true, then its converse is not necessarily true.

If a statement is a definition, then its converse is always true, and the two statements can be combined into an "if an only if" statement. $a \leftrightarrow b$

iff

"A figure is a triangle if and only if it is a polygon that has three sides."

True statement: If you are an astronaut, you are not more than six feet tall.

Hypothesis:

you are an astronaut

Conclusion:

you are not more than six feet tall

6. Write the converse of the statement.

If you are not more than six feet tall, then you are an astronaut.

7. Is the converse true?

no

8. Does it have the same meaning as the original statement?

no!

Definition: You have arachibutyrophobia iff you have the fear of peanut butter sticking to the roof of your mouth. ↑ if and only if

If the definition were represented in symbols as a ←→b and "a" represents "arachibutyrophobia," what does

14. ←→ represent?

if and only if

15. "b" represent?

the fear of peanut butter sticking to the roof of your mouth

In words, "a→b" for this definition is "If you have arachibutyrophobia, then you are afraid of peanut butter sticking to the roof of your mouth."

16. Write in words, "b→a" for this definition.

If you are afraid of . . . , then you have arachibutyrophobia

17. Is this sentence necessarily true? yes (definition)

(1) If it is your birthday, then you get some presents.

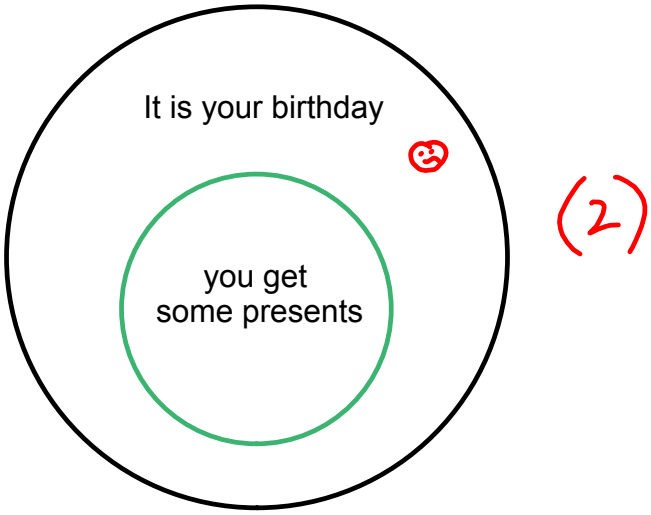
(2) Only if it is your birthday, do you get some presents.

(if you get some presents, then it must be your birthday)

24. Is the first sentence true for you?

25. Is the second sentence true for you?

27. Which sentence does this Euler Diagram illustrate?



(2)

2.3 – Direct Proof

A syllogism is an argument of the form

a → b

b → c

Therefore, a → c.

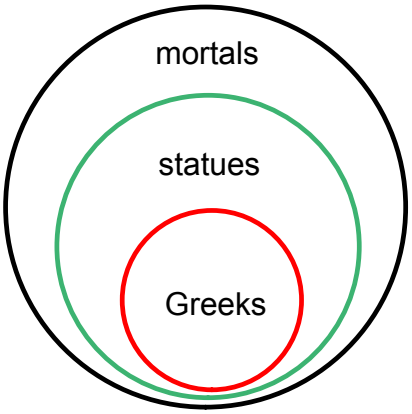
A syllogism is an example of a direct proof.

The statements a → b and b → c are called the premises of the argument.

a → c is called the conclusion of the argument, and is often considered to be a theorem.

A theorem is a statement that is proved by reasoning deductively from already accepted statements.

Syllogisms were discussed by the Greek philosopher Aristotle in the fourth century BC. Write the syllogism illustrated by the following Euler diagram.



If someone is Greek, then they are a statue.

If someone is a statue, then they are mortal

Therfore, if someone is Greek, then they are mortal.

6. If the premises of a syllogism are true, does it follow that its conclusion must be true? yes

7. If the premises of a syllogism are false, does it follow that its conclusion must be false? no (could be true or false)

"Admit one ridiculous premise and the rest follows." – Aristotle

If you live at the South Pole, you live in the Antarctic. P1

If you live in the Antarctic, you live where it is cold. P2

If you live where it is cold, you see a lot of penguins. P3

Therefore, if you live at the South Pole, you see a lot of penguins. C

What part of the second premise matches

8. the conclusion of the first premise? hypothesis

9. the hypothesis of the third premise? conclusion

$a \to b$
$b \to c$
$c \to d$
therefore $\therefore$ $a \to d$

10. Starting with a→b to represent the first premise, represent the entire argument in symbols.

11. Which premise is ridiculous (false)? P3

12. What does the fact that one premise is false indicate about the conclusion of the argument? conclusion may be true or false

If Captain Spaulding is in the jungle, there are too many cheetahs.

If there are too many cheetahs, Captain Spaulding can't play cards.

13. What conclusion follows from these premises?

If Captain Spaulding is in the jungle, then he can't play cards.

14. If the two premises are true, doe sit follow that the conclusion must be true?

true!

Write in the missing statements for the following proofs.

20. *Theorem:* If two hungry vultures took an airplane, they would be told that there is a limit of two carrion per passenger.

Proof:

If two hungry vultures took an airplane, they would want to take along some food.

> If they want to take along some food, they will try to carry on six dead racoons.

If they tried to carry on six dead raccoons, the flight attended would object.

> If the flight attendant objects, he or she will inform the vultures that there is a limit of two carcion per passenges.

21. *Theorem:* If a group of chess players checked into a hotel, the manager would say "I can't stand chess nuts boating in an open foyer."

Proof:

> If a group of chess players check into a hotel, then, they will stand in the lobby bragging about their tournament victories.

If they stood in the lobby bragging about their tournament victories, the manager would ask them to leave.

> If the manager asks them to leave, they will ask why

If they asked why, the manager would say "I can't stand chess nuts boasting in an open foyer."

HW #1 (due Friday, 11/7)
- Read Ch 1 & Ch 2
- **Ch 1 Review Problems pp. 36-38**
- Start working on Geometry badge on Khan Academy; make sure you've added me as a coach using code listed on brewermath.com!

Quiz #1 - Wednesday, 11/13
- Vocab
- Fill in the blank proofs

HW #2 (due Friday, 11/15)
- Read Ch 3 & Ch 4
- **Ch 2 Review Problems pp. 71-74**
- **Ch 3 Review Problems pp. 124-128**
- Khan Academy exercises under "Introduction to Euclidean Geometry"