2.4 – Indirect Proof

In an indirect proof, an assumption is made at the beginning that leads to a contradiction. The contradiction indicates that the assumption is false and the desired conclusion is true.

Direct versus Indirect proof of the theorem "If a, then d."

Direct Proof:

If a, then b.

If b, then c.

If c, then d.
Therefore, if a, then d.

Indirect Proof:

Suppose not d is true.

If not d, then e.

If e, then f,

And so on until we come to a contradiction.

Therefore, not d is false; so d is true.

List the assumption with which an indirect proof of each of the following statements would begin.

Example: If a tailor wants to make a coat last, he makes the pants first.

Answer: Suppose that he does not make the pants first.

4. If a teacher is cross-eyed, he has no control over his pupils.

Suppose that he does have control over his pupils.

5. If a proof is indirect, then it leads to a contradiction.

Suppose it does not lead to a contradiction.

In a book written in the 13th century on the shape of the earth, the author reasoned: **"If the earth were flat, the stars would rise at the same time for everyone, which they do not."**

11. What is the author trying to prove?

The earth is not flat

12. With what assumption does the author begin?

The earth is flat

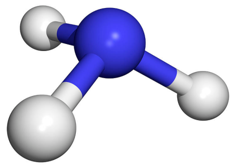
13. What is the contradiction?

stars don't rise at the same time for everyone

14. What does the contradiction prove about the author's beginning assumption?

it's false; therefore the earth is not flat

Write the missing statements in the indirect proof:



16. The ammonia molecule consists of three hydrogen atoms bonded to a nitrogen atom as shown in this figure.

The fact that chemists have found that each bond angle is 107° can be used to prove the following theorem.

Theorem: The atoms of an ammonia molecule are not coplanar.

Proof:

> Suppose that the atoms are coplanar.

If the atoms are coplanar, then the sum of the three bond angles is 360°.

If the sum of the three bond angles is 360°, then each angle is 120°.

> a contradiction to the fact that chemists have found the angles to be 107°.

Therefore, our assumption is false and

> the atoms are not coplanar.

19. A particular puzzle involves separating a set of twelve weights into two sets so that one set will exactly balance the other on a scale with two pans.

Consider this argument:

If a puzzle of this type has a solution, then the weights of the two sets will be equal.

a b If a, then b

If the weights of the two sets are equal, then each set will weigh half the total weight.

b c If b, then c

What conclusion follows from these two premises?

If a puzzle of this type has a solution, then each set will weigh half the total weight.

a c

∴ if a, then c.

20. **Write in the missing statements in the indirect proof about this puzzle:**

Theorem: If the sum of all of the weights is odd, then there is no solution.

Proof:

> suppose there is a solution.

If there is a solution, let the weights in one set add up to S.

If the weights in each set add up to S, then the weights in both sets add up to S+S=2S, an even number,

> a contradiction to the sum of weights being odd.

Therefore, our assumption is false and

> there is no solution.

21. At a sports banquet there are 100 famous athletes. Each one is either a football player or a basketball player. At least one is a football player. Given any two of the athletes, at least one is a basketball player. **How many of the athletes are football players, and how many are basketball players? Construct an indirect argument to explain your reasoning.**

Theorem: There are 99 basketball players and only one football player.

Proof: Suppose that there is more than one football player.

If there is more than one football player, then we could have a group of 2 football players, a contradiction to the fact that in every pair of athletes, at least one is a basketball player.

Therefore, our initial assumption is false and there must be only one football player.

2.5 – A Deductive System

To avoid circular definitions, mathematics leaves certain terms undefined.

Those which we have seen so far include: point, line, plane.

These undefined terms can be used to define other terms, for example,

Def: Points are collinear iff there is a line that contains all of them.

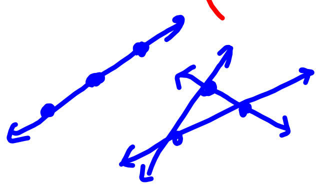
Def: Lines are concurrent iff they contain the same point.

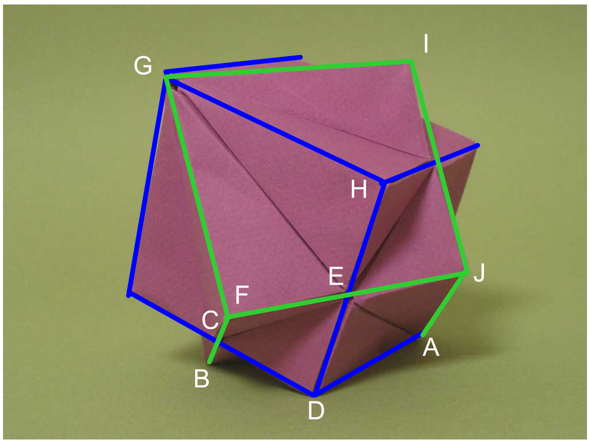
Just as it is impossible to define everything without going around in circles, it is impossible to prove everything. We leave some statements unproved, and use them as a basis for building proofs of other statements.

Def: A postulate is a statement that is assumed to be true without proof. (axiom)

Postulate 1: Two points determine a line.

Postulate 2: Three noncollinear points determine a plane.





Determine if the following statements are true or false:

19. Points B, C, and F are collinear.
true

20. Points B and C determine a line.
true

21. Points F, E, and J are coplanar.
true

22. Points F, E, and J determine a plane.
false (colinear)

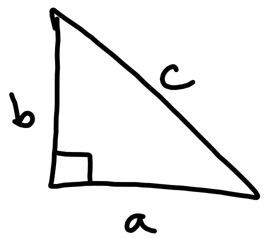
23. Points A, E, and G are collinear.
false

24. Points A, B, C, and J are coplanar.
true

25. Lines DH, FJ, and EG are concurrent.
true (point E)

2.6 - Some Famous Theorems of Geometry

The Pythagorean Theorem: The square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides.



$a^2 + b^2 = c^2$

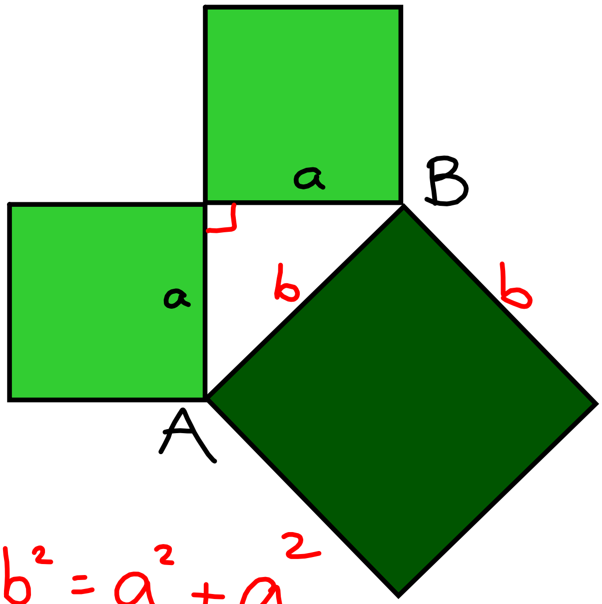
$\pi \approx 3.1415926535$
8979323846
2643383279
502884197
$1\ldots$

The Triangle Sum Theorem: The sum of the angles in a triangle is 180°.

Circle Theorems:

If the diameter of a circle is d, then its circumference is πd. $(2\pi r)$

If the radius of a circle is r, then its area is πr^2.



$b^2 = a^2 + a^2$

$b^2 = 2a^2$

34. The area of the light green square is

$a \times a = a^2$

35. The combined area of the two light green squares is

$a^2 + a^2 = 2a^2$

36. The area of the dark green square is

$b^2 = 2a^2$

41. "The area of a circle is half of the circumference multiplied by half of the diameter." ~ 6th century Indian astronomer Aryabhata

Is this true? Yes!

<u>Theorem</u>: $A = \left(\frac{1}{2}C\right) \times \left(\frac{1}{2}D\right)$

<u>Proof</u>: $\left(\frac{1}{2}C\right) \times \left(\frac{1}{2}D\right) =$

$= \frac{1}{2}(2\pi r) \times \frac{1}{2}(2r) =$

$= \pi r \times r =$

$= \pi r^2 = A$

HW #1 (submitted Friday, 11/7)
- Read Ch 1 & Ch 2
- **Ch 1 Review Problems pp. 36-38**
- Start working on Geometry badge on Khan Academy; make sure you've added me as a coach using code listed on brewermath.com!

Quiz #1 - Wednesday, 11/12
- Vocab
- Fill in the blank proofs

HW #2 (due Friday, 11/14)
- Read Ch 3 & Ch 4
- **Ch 2 Review Problems pp. 71-74**
- **Ch 3 Review Problems pp. 124-128**
- Khan Academy exercises:
  "Introduction to Euclidean geometry"
  "Angles and intersecting lines"