3.1 – Number Operations and Equality

Algebraic Postulates of Equality:

Reflexive Property: a=a (Any number is equal to itself.)

Substitution Property: If a=b, then a can be substituted for b in any expression.

Addition Property: If a=b, then a+c=b+c

Subtraction Property: If a=b, then a-c=b-c.

Multiplication Property: If a=b, then ac=bc.

Division Property: If a=b, then a/c=b/c. , $c \neq 0$

State the property of equality illustrated by each statement:

3. If c/d=π, then c=πd

multiplication property

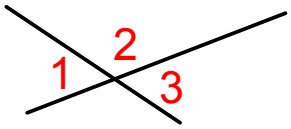
4. If ∠A+∠B+∠C=180° and ∠C=∠A+∠B, then ∠C+∠C=180°.

Substitution property

5. If 2∠C=180°, then ∠C=90°.

division property

This figure shows two lines intersecting to form several angles, three of which are numbered.



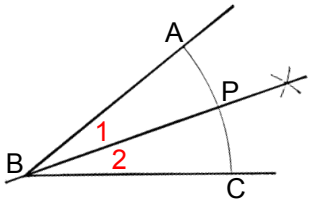
8. If ∠1+∠2=∠2+∠3, then ∠1=∠3. Why?

subtraction property

9. If ∠1=∠2 and ∠2=∠3, then ∠1=∠3. Why?

substitution property

This figure shows how we bisected an angle by using a straightedge and compass. Let's check the algebra to see that ∠1 is the size that we would expect.



11. If ∠ABC=∠1+∠2 and ∠1=∠2, then ∠ABC=∠1+∠1=2∠1. Why?

substitution (& simplification)

12. If ∠ABC=2∠1, then ∠1=(1/2) ∠ABC. Why?

division property

Quadratic formula

If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$.

37. What is the hypothesis of this theorem?

$ax^2+bx+c=0$

Name the postulate that is the reason for each of the following first three steps in its proof:

38. If $ax^2 + bx + c = 0$, then $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$.

division property

39. If $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$, then $x^2 + \frac{b}{a}x = -\frac{c}{a}$.

subtraction property

40. If $x^2 + \frac{b}{a}x = -\frac{c}{a}$, then $x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$.

addition property

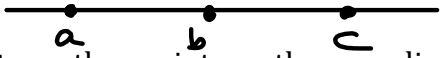
41. What kind of proof begins like this?

direct proof (syllogism)

3.2 – The Ruler and Distance

Postulate 3: The Ruler Postulate – The points on a line can be numbered so that positive number differences measure distance.

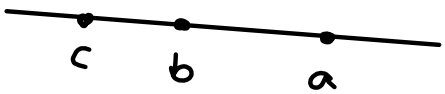
distance between a&b is $|a-b| = |b-a|$



Def: Betweenness of Points – A point is between two other points on the same line iff its coordinate is between their coordinates.

(More briefly, A-B-C iff a<b<c or a>b>c.)

B is between A & C



Theorem 1: The Betweenness of Points Theorem

If A-B-C, then AB+BC=AC

AB = distance between A & B

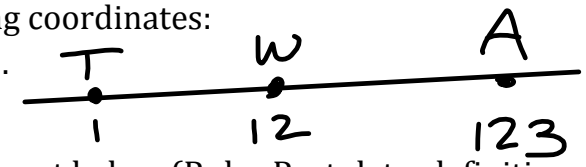
Proof for a<b<c case:

	Statements:	**Reasons:**
1.	A-B-C	The hypothesis.
2.	a<b<c	Definition of betweenness.
3.	AB=b-a and BC=c-b	Ruler Postulate.
4.	AB+BC=(b-a)+(c-b)=c-a	Addition (and simplifaction).
5.	AC=c-a	Ruler Postulate.
6.	AB+BC=AC	Substitution (steps 4 and 5).

$AB+BC = b-a+BC$

& substitution

Three points on a line have the following coordinates: point A, 123; point T, 1; and point W, 12.



Which idea is the reason for each statement below (Ruler Postulate, definition of betweenness of points, or Betweenness of Points Theorem)?

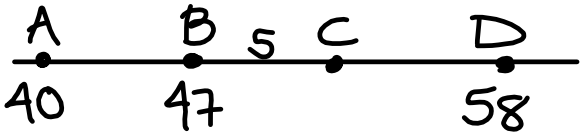
4. T-W-A because 1<12<123.

definition of betweenness of points (A-B-C iff a<b<c or a>b>c)

5. TW+WA=TA because T-W-A.

Betweenness of Points Theorem

Suppose point A is at coordinate 40, point B is at coordinate 47, distance BC is 5, and point D is at coordinate 58. Determine:



1. The total distance AD.

58 − 40 = 18 (Ruler postulate)

2. The coordinate of C.

47 + 5 = 52

C − 47 = 5 — Ruler Postulate

C = 47 + 5 — Addition

3. The distance CD.

58 − 52 = 6 (Ruler Postulate)

Because A-B-C, AB+BC=AC, or 7+5=12, according to the Betweenness of Points Theorem. Use this theorem to complete the statements:

1. 9. Because B-C-D, BC+CD = BD, or 5+ 6 = 11 .

2. 10. Because A-B-D, AB+ BD = AD, or 7+ 11 = 18 .

3. 11. Because A-C-D, AC+ CD = AD, or 12 + 6 = 18 .

Suppose AC=BD. Complete the statements:

38. Because A-B-C, AC= AB + BC (Betweenness of Points Theorem)

39. Because B-C-D, BD= BC + CD (Betweenness of Points Theorem)

40. Why is AB+BC=BC+CD?
Substitution

41. Why is AB=CD?
Subtraction

3.3 – The Protractor and Angle Measure

Postulate 4: The Protractor Postulate – The rays in a half-rotation can be numbered from 0 to 180 so that positive number differences measure angles.

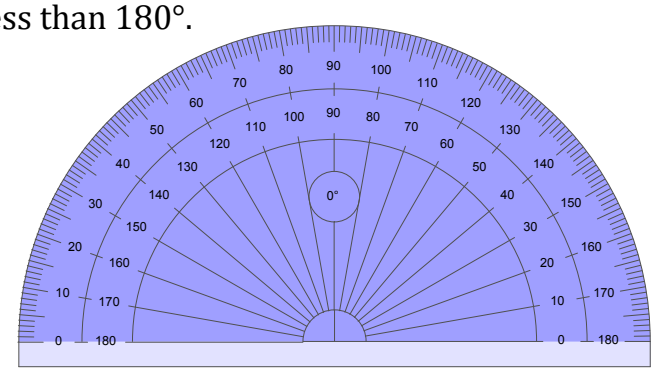
Definitions: An angle is

Acute iff it is less than 90°.

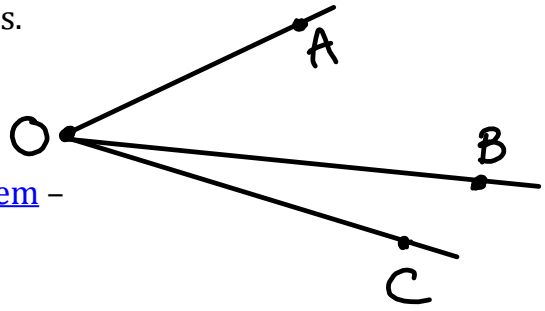
Right iff it is 90°.

Obtuse iff it is more than 90° but less than 180°.

Straight iff it is 180°.



Def: Betweenness of Rays – A ray is between two others in the same half-rotation iff its coordinate is between their coordinates.
(More briefly, OA-OB-OC iff a<b<c or a>b>c.)



Theorem 2: The Betweenness of Rays Theorem –
If OA-OB-OC, then ∠AOB+∠BOC=∠AOC.

Proof for a>b>c case:

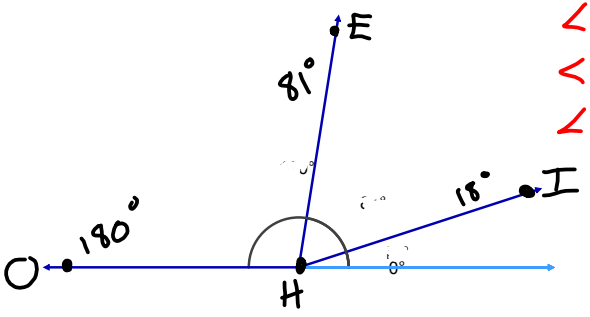
Statements:	**Reasons:**
1. OA-OB-OC	The hypothesis.
2. a>b>c	Definition of betweenness.
3. ∠AOB=a-b and ∠BOC=b-c	Protractor Postulate.
4. ∠AOB+∠BOC=(a-b)+(b-c)=a-c	Addition (and simplification).
5. ∠AOC=a-c	Protractor Postulate.
6. ∠AOB+∠BOC=∠AOC	Substitution (steps 4 and 5).

Three rays in a half-rotation have the following coordinates:
ray HE, 81; ray HI, 18; and ray HO, 180.

4. Which ray is between the other two (and why)?

HI - HE - HO because 18 < 81 < 180
(definition of betweenness of rays)

Use your protractor to draw a figure.



∠OHE
∠EHI
∠IHO = ∠OHI

5. Name and find the measures of the three angles formed by the rays.

because of the Betweenness of Rays Theorem,
Since HI-HE-HO,
∠OHE + ∠EHI = ∠OHI

HW #1 (submitted Friday, 11/7)
- Read Ch 1 & Ch 2
- **Ch 1 Review Problems pp. 36-38**
- Start working on Geometry badge on Khan Academy; make sure you've added me as a coach using code listed on brewermath.com!

Quiz #1 - Wednesday, 11/12
- Vocab
- Fill in the blank proofs

HW #2 (due Friday, 11/14)
- Read Ch 3 & Ch 4
- **Ch 2 Review Problems pp. 71-74**
- **Ch 3 Review Problems pp. 124-128**
- Khan Academy exercises:
  "Introduction to Euclidean geometry"
  "Angles and intersecting lines"