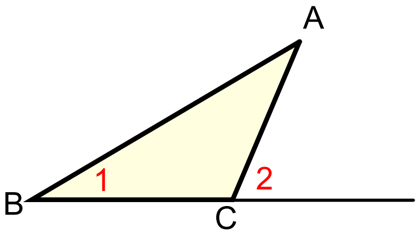
The "Three Possibilities" Property: either a>b, a=b, or a<b
The Transitive Property: If a>b and b>c, then a>c
The Addition Property: If a>b, then a+c>b+c
The Subtraction Property: If a>b, then a-c>b-c
The Multiplication Property: If a>b and c>0, then ac>bc
The Division Property: If a>b and c>0, then a/c>b/c

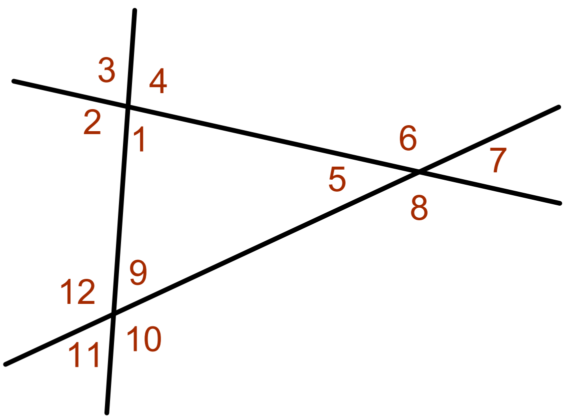
The Addition Theorem of Inequality: If a>b and c>d, then a+c>b+d

The "Whole Greater than Part" Theorem: If a>0, b>0, and a+b=c, then c>a and c>b

Def: An exterior angle of a triangle is an angle that forms a linear pair with an angle of the triangle.
In ΔABC, exterior ∠2 forms a linear pair with ∠ACB.
The other two angles of the triangle, ∠1 (∠B) and ∠A are called remote interior angles with respect to ∠2.



Theorem 12: The Exterior Angle Theorem
An Exterior angle of a triangle is greater than either remote interior angle.



Find each of the following sums.

26. ∠1+∠2+∠3+∠4

360°

27. ∠1+∠2+∠3+∠4+∠5+∠6+∠7+∠8+∠9+∠10+∠11+∠12

$\boxed{1080°}$ = 360° × 3

28. ∠1+∠5+∠9

180° (triangle sum theorem)

29. ∠3+∠7+∠11

180° (vertical ∠s are equal & substitution)

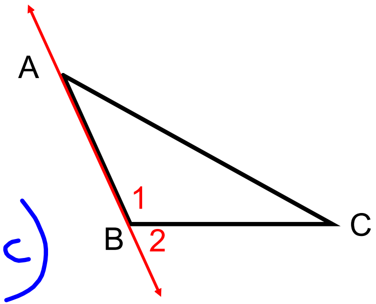
30. ∠2+∠4+∠6+∠8+∠10+∠12

1080° − 180° − 180° = $\boxed{720°}$

31. What does the result in exercise 30 indicate about the sum of the exterior angles of a triangle?

always = 720°

After proving the Exterior Angle Theorem, Euclid proved that, in any triangle, the sum of any two angles is less than 180°. Prove that, in ΔABC, ∠A+∠B<180° by giving a reason for each of the following statements.



39. Draw line AB.

2 points define a line

40. ∠2 is an exterior angle of ΔABC.

def'n of exterior ∠'s
(∠2 forms a linear pair w/ ∠ABC)

41. ∠1 and ∠2 are supplementary.

∠'s in a linear pair are supplementary

42. ∠1+∠2=180°.

supplementary ∠'s sum to 180°

43. ∠2>∠A

exterior angle theorem (exterior ∠ is greater than either remote interior ∠)

44. ∠1+∠2>∠1+∠A

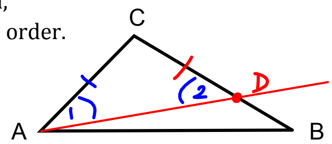
Addition

45. 180°>∠1+∠A, so ∠1+∠A<180°

Substitution (#42 & 44)

5.3 Triangle Side and Angle Inequalities

Theorem 13: If two sides of a triangle are unequal, the angles opposite them are unequal in the same order.

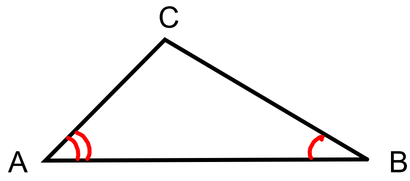


Given: ΔABC with BC>AC
Prove: ∠A>∠B

Proof:

Statements	**Reasons**
1. ΔABC with BC>AC	Given
2. Choose D on CB so that CD=CA	Ruler Postulate & compass construction of equal line segments
3. Draw AD	2 points define a line
4. ∠1=∠2	If 2 sides in a Δ are equal, the angles opposite them are equal
5. ∠CAB=∠1+∠DAB	Betweenness of Rays Theorem
6. ∠CAB>∠1	Whole greater than Part
7. ∠CAB>∠2	Substitution (# 4 & # 6)
8. ∠2>∠B	∠2 is an exterior ∠ for ΔADB & hence is greater than either remote interior ∠ in ΔADB
9. ∠CAB>∠B	Transitivity

Theorem 14: If two angles of a triangle are unequal, the sides opposite them are unequal in the same order.



Given: ΔABC with ∠A>∠B
Prove: BC>AC

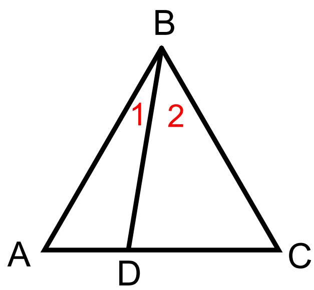
Proof:

Statements **Reasons**

Suppose that BC is not longer than AC

4. Then either BC=AC or BC<AC — 5. 3 Possibilities property
6. If BC=AC, then ∠A=∠B — 7. If 2 sides of a triangle are equal, then the angles opposite them are equal
8. This contradicts the hypothesis (given) that ∠A>∠B
9. If BC<AC, then ∠A<∠B — 10. If two sides of a Δ are unequal, the angles opposite them are unequal in the same order
11. This also contradicts the hypothesis that ∠A>∠B
12. Therefore, what we suppose is false and BC>AC.

Given: ΔABC is equilateral.
Prove: BD>DC

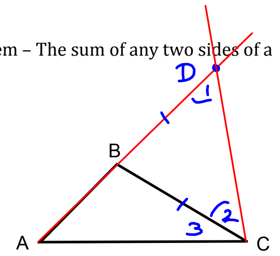


Proof:

Statements	Reasons
45. ∠C=∠ABC	equilateral Δ's are also equiangular
46. ∠ABC=∠1+∠2	Betweenness of Rays Theorem
47. ∠ABC>∠2	Whole greater than part
48. ∠C>∠2.	Substitution (#45 & #47)
49. BD>DC	If 2 ∠'s in a Δ are unequal, the sides opposite them are unequal in the same order

5.4 The Triangle Inequality Theorem

Theorem 15: The Triangle Inequality Theorem – The sum of any two sides of a triangle is greater than the third side.



Given: ABC is a triangle

Prove: AB+BC>AC

Proof:

Statements	Reasons
1. ABC is a triangle	Given
2. Draw line AB	2 points define a line
3. Choose D beyond B on line AB so that BD=BC	Ruler Postulate & compass construction of equal line segments
4. Draw CD	2 points define a line
5. ∠1=∠2	If two sides of a Δ are equal, the angles opposite them are equal
6. ∠ACD=∠2+∠3	Betweenness of Rays Theorem
7. ∠ACD>∠2	Whole is Greater than Part
8. ∠ACD>∠1	Substitution (#5 & #7)
9. In ΔACD, AD>AC	If 2 ∠'s in a Δ are unequal the sides opposite them are unequal in the same order
10. AB+BD=AD	Betweenness of Points Theorem
11. AB+BD>AC	Substitution (#9 & #10)
12. AB+BC>AC	Substitution (#3 & #11)

SAT Problem:

If x is an integer and 2<*x*<7, how many different triangles are there with sides of lengths 2, 7, and *x*?

15. Could *x*=3? Why or why not?

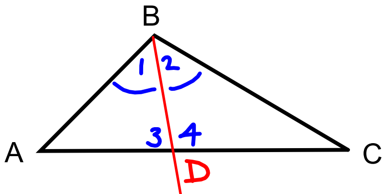
NO! $2+3 \not> 7$

16. What do you think is the answer to the problem? Explain.

only one! 2, 6, 7

Heron's Proof of the Triangle Inequality

Given: ABC is a triangle.
Prove: AB+BC>AC



Proof:

Statements	**Reasons**
24. Let BD bisect ∠ABC	every angle has a unique bisector
25. ∠1=∠2	angle bisectors divide an angle into two equal angles
26. ∠3>∠2 and ∠4>∠1	exterior ∠'s are greater than either remote interior angle
27. ∠3>∠1 and ∠4>∠2	Substitution (#25 & #26)
28. AB>AD and BC>DC	If 2 ∠'s in a Δ are unequal, the sides opposite them are unequal in same order
29. AB+BC>AD+DC	Addition Theorem of Inequality
30. AD+DC=AC	Betweeness of Points Theorem
31. AB+BC>AC	Substitution (#29 & 30)

HW #1 (submitted Friday, 11/7)
- Read Ch 1 & Ch 2
- Ch 1 Review Problems pp. 36-38
- Start working on Geometry badge on Khan Academy

HW #2 (submitted Friday, 11/14)
- Read Ch 3 & Ch 4
- Ch 2 Review Problems pp. 71-74
- Ch 3 Review Problems pp. 124-128
- Khan Academy exercises:
  "Introduction to Euclidean geometry"
  "Angles and intersecting lines"

HW #3 (submitted Friday, 11/21)
- Read Ch 4 & Ch 5
- Ch 4 Review Problems pp.176-180
- Khan Academy exercises: "Congruence"

HW #4 (due Friday, 12/5)
- Read Ch 5 & Ch 6
- **Ch 5 Review Problems pp. 206-209**
- Start working on Ch 6 Review Problems (not due until Fri. 12/12)
- Work toward mastery of practiced Khan Academy exercises in "Introduction to Euclidean Geometry," "Angles and Intersecting Lines," and "Congruence"