

Theorem 17: Equal corresponding angles mean that lines are parallel.

Corollary 1: Equal alternate interior angles mean that lines are parallel.

Corollary 2: Supplementary interior angles on the same side of a transversal mean that lines are parallel.

Corollary 3: In a plane, two lines perpendicular to a third line are parallel.

The Parallel Postulate – Through a point not on a line, there is exactly one line parallel to the given line.

Theorem 18: In a plane, two lines parallel to a third line are parallel to each other.

Theorem 19: Parallel lines form equal corresponding angles.

Corollary 1: Parallel lines form equal alternate interior angles.

Corollary 2: Parallel lines form supplementary interior angles on the same side of a transversal.

Corollary 3: In a plane, a line perpendicular to one of two parallel lines is also perpendicular to the other.

Theorem 20: The Triangle Sum Theorem – The sum of the angles of a triangle is 180° .

Corollary 1: If two angles of one triangle are equal to two angles of another triangle, the third angles are equal.

Corollary 2: The acute angles of a right triangle are complementary.

Corollary 3: Each angle of an equilateral triangle is 60° .

Theorem 21: An exterior angle of a triangle is equal to the sum of the remote interior angles.

Theorem 22: The AAS Theorem – If two angles and the side opposite one of them in one triangle are equal to the corresponding parts of another triangle, the triangles are congruent.

Theorem 23: The HL Theorem – If the hypotenuse and a leg of one right triangle are equal to the corresponding parts of another right triangle, the triangles are congruent.



7.1 - Quadrilaterals

Def: A **diagonal** of a polygon is a line segment that connects any two nonconsecutive vertices.

Theorem 24: **The sum of the angles of a quadrilateral is 360° .**

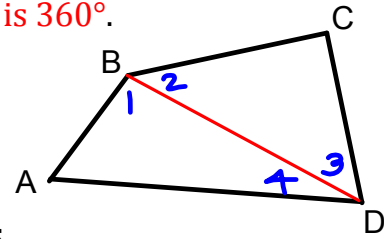
Given: ABCD is a quadrilateral.

Prove: $\angle A + \angle B + \angle C + \angle D = 360^\circ$

Proof:

Statements

1. Draw BD
2. $\angle A + \angle 1 + \angle 4 = 180^\circ$ and $\angle 2 + \angle 3 + \angle C = 180^\circ$
3. $\angle A + \angle 1 + \angle 4 + \angle 2 + \angle 3 + \angle C = \del{180^\circ} 360^\circ$
4. $\angle 1 + \angle 2 = \angle ABC$ and $\angle 3 + \angle 4 = \angle CDA$
5. $\angle A + \angle ABC + \angle C + \angle CDA = 360^\circ$
 $(\angle B) \quad (\angle D)$



Reasons

- 2 points define a line
- Triangle sum theorem
- Addition (& simplification)
- Betweenness of Rays Theorem
- Substitution (# 3 & # 4)

Def: A **rectangle** is a quadrilateral each of whose angles is a right angle.

Corollary to Theorem 24: A quadrilateral is equiangular iff it is a rectangle.

Given: ABCD with $\angle A = \angle B = \angle C = \angle D$.
 Prove: ABCD is a rectangle. \Rightarrow (statement)

Given: ABCD is a rectangle.
 Prove: $\angle A = \angle B = \angle C = \angle D$. \Leftarrow (converse)

Proof of \Rightarrow
statements

1. ABCD w/ $\angle A = \angle B = \angle C = \angle D$
2. $\angle A + \angle B + \angle C + \angle D = 360^\circ$
3. $\angle A + \angle A + \angle A + \angle A = 360^\circ$
 $4\angle A = 360^\circ$
4. $\angle A = 90^\circ$
5. $\angle B = \angle C = \angle D = 90^\circ$
6. $\angle A, \angle B, \angle C, \angle D$ are right angles
7. ABCD is a rectangle

Reasons

- Given
- Quadrilateral Sum Theorem
- Substitution (#1 & 2) & Simplification
- Division
- Substitution (#1 & 4)
- Right angles measure 90°
- Quadrilaterals w/ 4 right angles are rectangles.

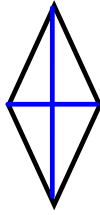
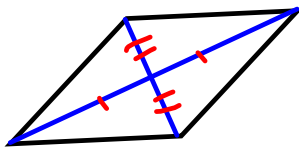
Proof of \Leftarrow
statements

1. ABCD is rectangle
2. $\angle A, \angle B, \angle C, \angle D$ are right \angle 's
3. $\angle A = \angle B = \angle C = \angle D$

Reasons

- Given
- Rectangles have 4 right angles
- Right angles are equal

Each of the figures below is a rhombus.



What seems to be true about

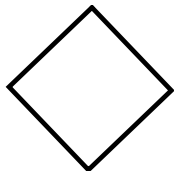
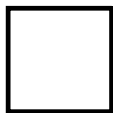
19. the sides of a rhombus?

*all sides are same length
opposite sides are parallel*

20. the diagonals of a rhombus?

*perpendicular
lines of symmetry of each other (& rhombus itself)
bisect each other*

Each of the figures below is a square.



What property do you think squares have in common with

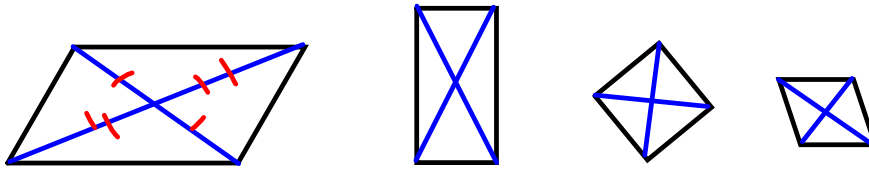
21. rectangles

all angles are right angles

22. rhombuses?

*parallel sides
sides all same length
2 diagonals w/ all same properties of rhombi*

Each of the figures below is a parallelogram.



What seems to be true about

23. The opposite sides of a parallelogram?

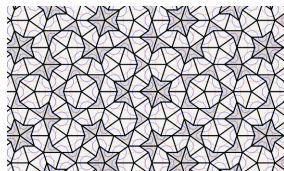
they are parallel

24. The opposite angles of a parallelogram?

equal

25. The diagonals of a parallelogram?

bisect each other



28. Which Penrose tile is convex?

kite

29. Find the measure of $\angle BCD$.

$$360^\circ - 3(72^\circ) = 144^\circ$$

30. Draw AC and CE.

The figure is drawn so that $AB=BE=ED=DA$ and $CB=CE=CD$.

31. How do you know that $\triangle ADC \cong \triangle ABC$ and $\triangle EDC \cong \triangle EBC$?

SAS SSS

Find the measures of the rest of the angles.

32. What do all four triangles in the figure have in common?

isosceles

33. What seems to be true about points A, C, and E?

colinear

34. Why?

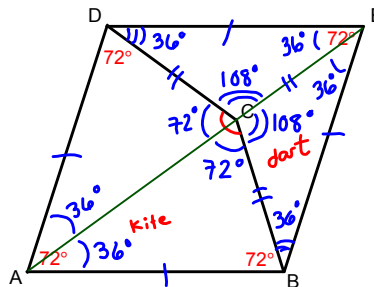
supplementary \angle 's on either side

35. Does the figure appear to have line symmetry? Why or why not?

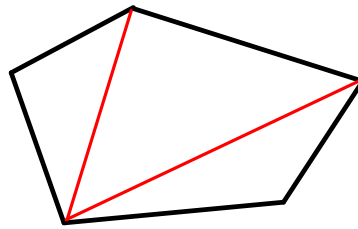
yes, along AE

36. If a quadrilateral is equilateral, does it follow that it is also equiangular? Why or why not?

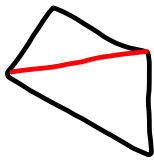
no! only true for square



In the figure below, a pentagon has been divided into triangles by the diagonals from one vertex.

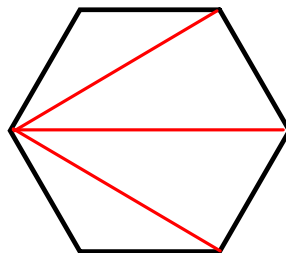


38. How many sides does a pentagon have, **5**
 how many diagonals were drawn, **2**
 and how many triangles were formed? **3**



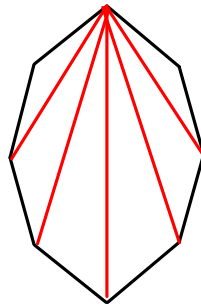
Recall, for 4-gon,
 4 sides
 1 diagonal
 2 Δ 's

39. Draw a hexagon and the diagonals from one vertex.



40. How many sides does a hexagon have, **6**
 how many diagonals did you draw, **3**
 and how many triangles were formed? **4**

41. Draw the diagonals from one vertex for the given figure.



42. How many sides does the polygon have, **8 (octagon)**
 how many diagonals did you draw, **5**
 and how many triangles were formed? **6**

In general, if a polygon has n sides, in terms of n ,

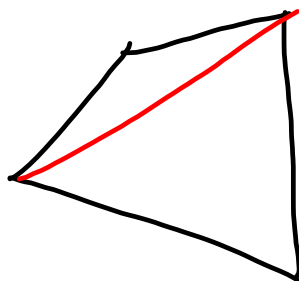
43. how many diagonals can be drawn from one vertex?

$$n - 3$$

44. how many triangles do these diagonals form?

$$n - 3 + 1 = n - 2$$

45. Show that your answers are correct for a quadrilateral.

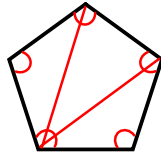


$$n = 4 \text{ sides}$$

$$n - 3 = 4 - 3 = 1 \text{ diagonal}$$


$$n - 2 = 4 - 2 = 2 \text{ triangles}$$

The figure below suggests that the sum of the angles of a pentagon is $3 \times 180^\circ = 540^\circ$.



If the pentagon is equiangular, then each angle is $540^\circ/5 = 108^\circ$.

46. What is the sum of the angles of a hexagon

 $4 \times 180^\circ = 720^\circ$

47. If the hexagon is equiangular, how large is each angle?

$720^\circ / 6 = 120^\circ$

48. What is the sum of the angles of an octagon?

$6 \times 180^\circ = 1080^\circ$

49. If the octagon is equiangular, how large is each angle?

$1080^\circ / 8 = 135^\circ$

50. What, in terms of n , is the sum of the angles of an n -gon?

$(n-2)(180^\circ)$

51. If the n -gon is equiangular, how large is each angle in terms of n ?

$\frac{(n-2)(180^\circ)}{n}$

7.2 - Parallelograms and Point Symmetry

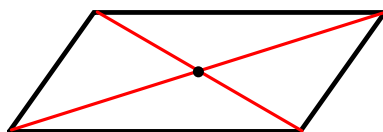
Def: A parallelogram is a quadrilateral whose opposite sides are parallel.



A figure has point symmetry if it looks exactly the same when it is rotated about a point.

Def: Two points are symmetric with respect to a point iff it is the midpoint of the line segment joining them.

Parallelograms have point symmetry about the point in which their diagonals intersect.



[HW #1](#) (submitted Friday, 11/7)

- [Ch 1 Review Problems pp. 36-38](#)
- Start working on Geometry badge on [Khan Academy](#)

[HW #2](#) (submitted Friday, 11/14)

- [Ch 2 Review Problems pp. 71-74](#)
- [Ch 3 Review Problems pp. 124-128](#)
- Khan Academy exercises: "[Introduction to Euclidean geometry](#)," "[Angles and intersecting lines](#)"

[HW #3](#) (submitted Friday, 11/21)

- [Ch 4 Review Problems pp.176-180](#)
- Khan Academy exercises: "[Congruence](#)"

[HW #4](#) (submitted Friday, 12/5)

- [Ch 5 Review Problems pp. 206-209](#)
- Work toward [mastery of practiced Khan Academy exercises](#) in "Introduction to Euclidean Geometry," "Angles and Intersecting Lines," and "Congruence"

[HW #5 \(due Friday, 12/12\)](#)

- **[Ch 6 Review Problems pp. 250-254](#)**
- Start working on Ch 7 Review Problems pp. 292-295