Notes Re: Test #2

Every statement (theorem) to be proved can be restated as an "If A, then B" statement or as an "A if and only if B" statement, in which case, both statements "If A, then B" and "If B, then A" need to be proved.

The hypothesis (A in "If A, then B") is what is <u>given</u> in the theorem, and is typically the first statement in a proof, with the reason being "given."

The conclusion (B in "If A, then B") is what you are trying to prove, and should be the last statement in a proof. This expression can NEVER be used as a reason in its own proof.

Restate "The sum of the angles of a quadrilateral is 360°"

AS: "If ABCD is a quadrilateral, then ∠A+∠B+∠C+∠D=360°"

Given
To Prove

Theorem 17: Equal corresponding angles mean that lines are parallel.
Corollary 1: Equal alternate interior angles mean that lines are parallel.
Corollary 2: Supplementary interior angles on the same side of a transversal mean that lines are parallel.
Corollary 3: In a plane, two lines perpendicular to a third line are parallel.

The Parallel Postulate – Through a point not on a line, there is exactly one line parallel to the given line.
Theorem 18: In a plane, two lines parallel to a third line are parallel to each other.

Theorem 19: Parallel lines form equal corresponding angles.
Corollary 1: Parallel lines form equal alternate interior angles.
Corollary 2: Parallel lines form supplementary interior angles on the same side of a transversal.
Corollary 3: In a plane, a line perpendicular to one of two parallel lines is also perpendicular to the other.

Theorem 20: The Triangle Sum Theorem – The sum of the angles of a triangle is 180°.
Corollary 1: If two angles of one triangle are equal to two angles of another triangle, the third angles are equal.
Corollary 2: The acute angles of a right triangle are complementary.
Corollary 3: Each angle of an equilateral triangle is 60°.

Theorem 21: An exterior angle of a triangle is equal to the sum of the remote interior angles.

Theorem 22: The AAS Theorem – If two angles and the side opposite one of them in one triangle are equal to the corresponding parts of another triangle, the triangles are congruent.

Theorem 23: The HL Theorem – If the hypotenuse and a leg of one right triangle are equal to the corresponding parts of another right triangle, the triangles are congruent.

Def: A diagonal of a polygon is a line segment that connects any two nonconsecutive vertices.

Theorem 24: The sum of the angles of a quadrilateral is 360°.

Def: A rectangle is a quadrilateral each of whose angles is a right angle.

Corollary to Theorem 24: A quadrilateral is equiangular iff it is a rectangle.

In general, if a polygon has n sides, in terms of n,
- n-3 diagonals can be drawn from one vertex
- these diagonals form n-2 triangles
- the sum of the angles of an n-gon is (n-2)*180°
- If the n-gon is equiangular, each angle measures (n-2)*180°/n

Def: A parallelogram is a quadrilateral whose opposite sides are parallel.

A figure has point symmetry if it looks exactly the same when it is rotated about a point.

Def: Two points are symmetric with respect to a point iff it is the midpoint of the line segment joining them.

Parallelograms have point symmetry about the point in which their diagonals intersect.

Theorem 25: The opposite sides and angles of a parallelogram are equal.
Theorem 26: The diagonals of a parallelogram bisect each other.
Theorem 27: A quadrilateral is a parallelogram if its opposite sides are equal.
Theorem 28: A quadrilateral is a parallelogram if its opposite angles are equal.
Theorem 29: A quadrilateral is a parallelogram if two opposite sides are both parallel and equal.
Theorem 30: A quadrilateral is a parallelogram if its diagonals bisect each other.

7.4 – Rectangles, Rhombuses, and Squares

Def: A square is a quadrilateral all of whose sides and angles are equal.
Every square is a rhombus.

Def: A rhombus is a quadrilateral all of whose sides are equal.

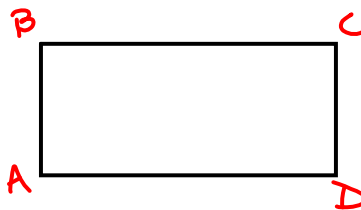Theorem 31: All rectangles are parallelograms.
Given: ABCD is a rectangle.
Prove: ABCD is a parallelogram.

Proof:

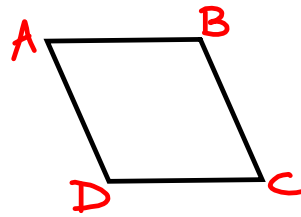| Statements	Reasons
1. ABCD is a rectangle	Given
2. ∠A, ∠B, ∠C, and ∠D are right angles	All angles in a rectangle are right angles
3. ∠B = ∠D and ∠A = ∠C	All right angles are equal
4. ABCD is a parallelogram	A quadrilateral with both pairs of opposite angles equal is a parallelogram.

<u>Theorem 32</u>: All rhombuses are parallelograms.

Given: ABCD is a rhombus.

Prove: ABCD is a parallelogram.

Proof

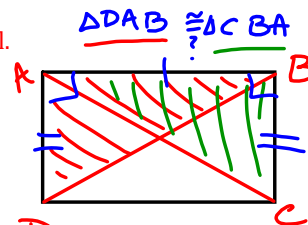| Statements	Reasons
1. ABCD is a rhombus	Given
2. AB = BC = CD = DA	All sides of a rhombus are equal
3. ABCD is a parallelogram	A quadrilateral is a parallelogram if its opposite sides are equal

<u>Theorem 33</u>: The diagonals of a rectangle are equal.
Given: ABCD is a rectangle.

Prove: AC=BD.

ΔDAB ≅ ΔCBA

Proof

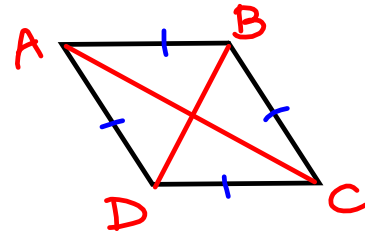| Statements	Reasons
1. ABCD is a rectangle	Given
2. ∠A, ∠B, ∠C, and ∠D are all right angles	All angles in a rectangle are right angles
3. Draw AC & BD	2 points define a line
4. ∠A = ∠B	All right angles are equal
5. AB = AB	reflexive
6. ABCD is a parallelogram	All rectangles are parallelograms
7. DA = CB	Opposite sides of a parallelogram are equal
8. ΔPAB ≅ ΔCBA	SAS congruence
9. AC = BD	Corresponding parts of congruent triangles are equal

4

Theorem 34: The diagonals of a rhombus are perpendicular.

Given: ABCD is a rhombus.

Prove: AC   BD.

Proof

statements	Reasons
1. ABCD is a rhombus	Given
2. Draw AC & BD	2 points define a line
3. AB = BC = CD = DA	all sides of a rhombus are equal
4. AC ⊥ BD	2 points equidistant from endpoints of a line segment form its perpendicular bisector

Regular dodecagon

1. How many sides does a dodecagon have?

   12
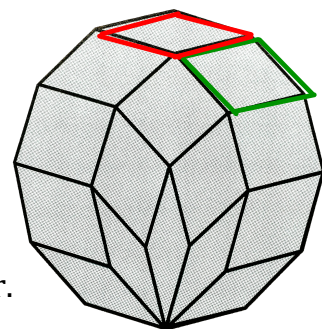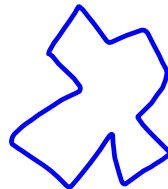
A regular polygon is one that is equilateral and equiangular.

2. How many regular quadrilaterals do there seem to be in the figure?

   3

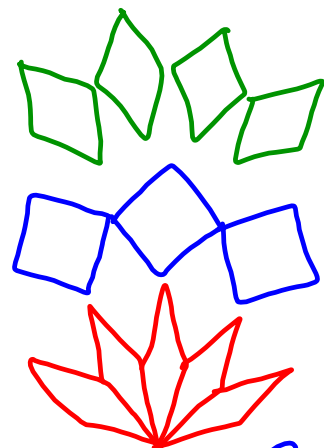3. What is a regular quadrilateral called?

   Square

4. How many rectangles do there seem to be in the figure?

   3

5. How many rhombuses are in the figure?

   15

6. How many different shapes of rhombuses does the figure seem to contain?   3

7.5 – Trapezoids

Def: A <u>trapezoid</u> is a quadrilateral that has exactly one pair of parallel sides.

The parallel sides are called the <u>bases</u> of the trapezoid, and the non-parallel sides are called its <u>legs</u>. The pairs of angles that include each base are called <u>base angles</u>.

∠A & ∠B are a pair of base angles
∠D & ∠C are a pair of base angles
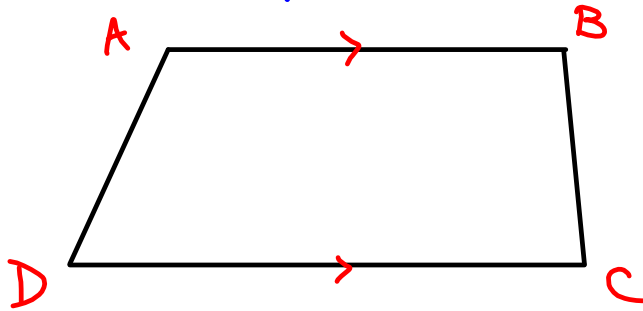
AB & DC are bases

AD & BC are legs

Def: An <u>isosceles trapezoid</u> is a trapezoid whose legs are equal.

<u>Theorem 35</u>: The base angles of an isosceles trapezoid are equal.
Given: ABCD is an isosceles trapezoid with bases AB and DC.
Prove: ∠A=∠B and ∠D=∠C

Proof

Statements	Reasons
1. ABCD is an isosceles trapezoid w/ bases AB and DC	Given
2. AD = BC	Legs of an isosceles trapezoid are equal
3. AB ∥ DC	Bases of a trapezoid are parallel
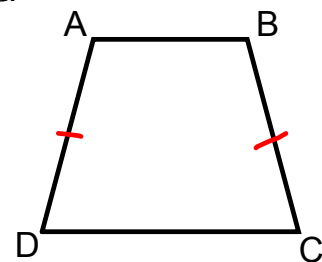
HW #1 (submitted Friday, 11/7)
- Ch 1 Review Problems pp. 36-38
- Start working on Geometry badge on Khan Academy

HW #2 (submitted Friday, 11/14)
- Ch 2 Review Problems pp. 71-74
- Ch 3 Review Problems pp. 124-128
- Khan Academy exercises:"Introduction to Euclidean geometry," "Angles and intersecting lines"

HW #3 (submitted Friday, 11/21)
- Ch 4 Review Problems pp.176-180
- Khan Academy exercises:"Congruence"

HW #4 (submitted Friday, 12/5)
- Ch 5 Review Problems pp. 206-209
- Work towardmastery of practiced Khan Academy exercisesin "Introduction to Euclidean Geometry," "Angles and Intersecting Lines," and "Congruence"

HW #5 (submitted Monday, 12/15)
- Ch 6 Review Problems pp. 250-254

HW #6 (due Friday, 01/09)
- Ch 7 Review Problems pp. 292-295
- Khan Academy exercises: **anything that has been recommended by me!**