

Theorem 24: The sum of the angles of a quadrilateral is 360° .

Def: A rectangle is a quadrilateral each of whose angles is a right angle.

Corollary to Theorem 24: A quadrilateral is equiangular iff it is a rectangle.

Def: A parallelogram is a quadrilateral whose opposite sides are parallel.

Theorem 25: The opposite sides and angles of a parallelogram are equal.

Theorem 26: The diagonals of a parallelogram bisect each other.

Theorem 27: A quadrilateral is a parallelogram if its opposite sides are equal.

Theorem 28: A quadrilateral is a parallelogram if its opposite angles are equal.

Theorem 29: A quadrilateral is a parallelogram if two opposite sides are both parallel and equal.

Theorem 30: A quadrilateral is a parallelogram if its diagonals bisect each other.

Def: A square is a quadrilateral all of whose sides and angles are equal. Every square is a rhombus.

Def: A rhombus is a quadrilateral all of whose sides are equal.

Theorem 31: All rectangles are parallelograms.

Theorem 32: All rhombuses are parallelograms.

Theorem 33: The diagonals of a rectangle are equal.

Theorem 34: The diagonals of a rhombus are perpendicular.

Def: A regular polygon is one that is equilateral and equiangular.

Def: A trapezoid is a quadrilateral that has exactly one pair of parallel sides. The parallel sides are called the bases of the trapezoid, and the non-parallel sides are called its legs. The pairs of angles that include each base are called base angles.

Def: An isosceles trapezoid is a trapezoid whose legs are equal.

Theorem 35: The base angles of an isosceles trapezoid are equal.

Theorem 36: The diagonals of an isosceles trapezoid are equal.

If a quadrilateral is a trapezoid, then its diagonals cannot bisect each other.

Def: A midsegment of a triangle is a line segment that connects the midpoints of two of its sides.

Theorem 37: The Midsegment Theorem –

A midsegment of a triangle is parallel to the third side and half as long.

Def: A transformation is a one-to-one correspondence between two sets of points.

A translation slides an object a certain distance without turning it.

A reflection flips an object over a mirror line.

A rotation turns an object a certain number of degrees about a fixed point.

A dilation enlarges or reduces the size of an object.

Def: an isometry is a transformation that preserves distance and angle measure.

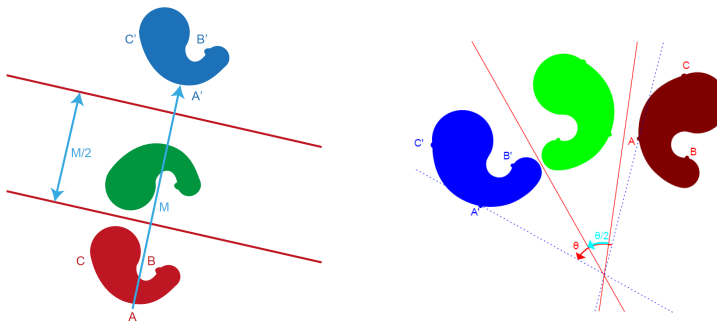
Translations, reflections, and rotations are all examples of isometries, but dilations are not.

Def: The reflection of point P through line l is P itself if P lies on l . Otherwise, it is the point P' such that l is the perpendicular bisector of PP'.

Construction 8: To reflect a point through a line.

Def: A translation is the composite of two successive reflections through parallel lines.

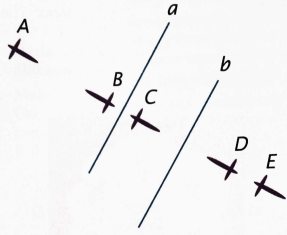
The distance between a point of the original figure and its translation image is called the *magnitude* of the translation.



Def: A rotation is the composite of two successive reflections through intersecting lines.

The point in which the lines intersect is the *center* of rotation, and the measure of the angle through which a point of the original figure turns to coincide with its rotation image is called the *magnitude* of the rotation.

In the figure below, $a \parallel b$ and birds A, B, D, and E are reflection images of bird C through either or both of the lines.



Which bird is the reflection image of

- 31. bird C through a ?
- 32. bird B through b ?
- 33. bird C through b ?
- 34. bird D through a ?

Which bird is the image of bird C as a result of successive reflections through

- 35. a and b ?
- 36. b and a ?

37. What transformation do exercises 35 and 36 illustrate?

31. B

32. E

33. D

34. A

35. E

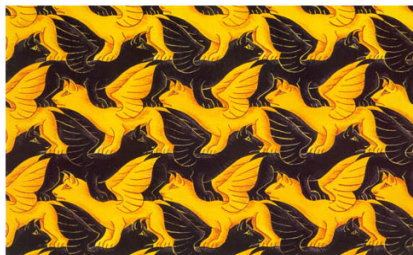
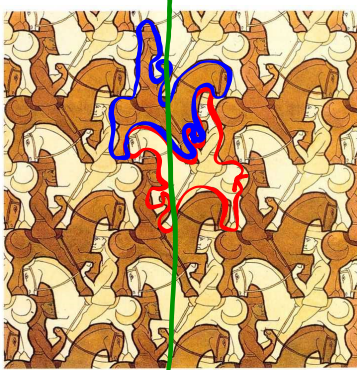
36. A

} 37 translations

8.3 - Isometries and Congruence

Def: Two figures are **congruent** if there is an isometry such that one figure is the image of the other.

Def: A **glide reflection** is the composite of a translation and a reflection in a line parallel to the direction of the translation.



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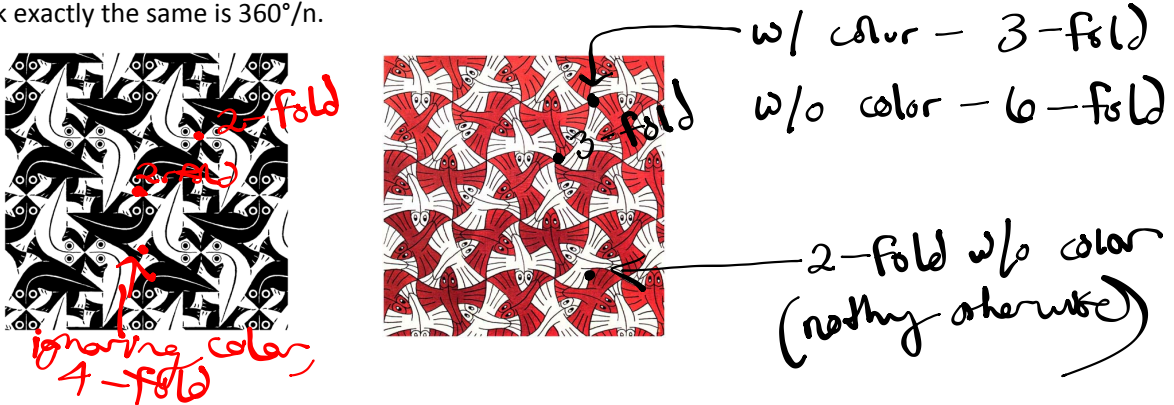


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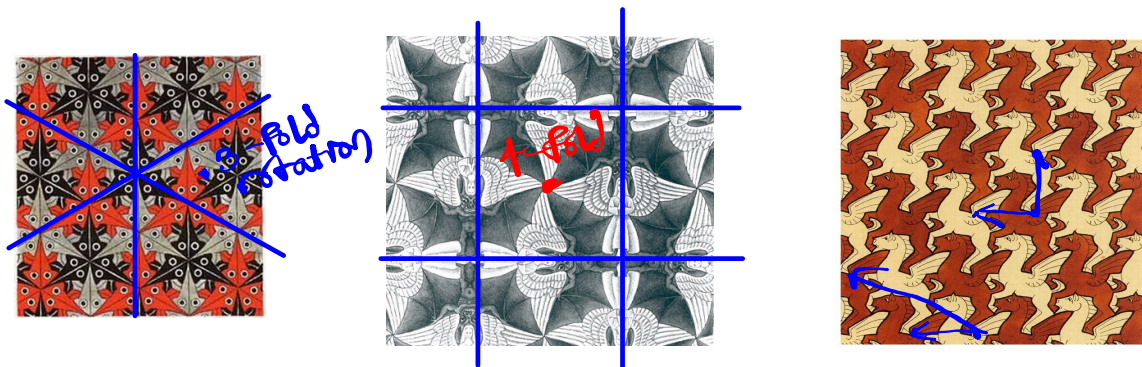
8.4 - Transformations and Symmetry

Def: A figure has rotation symmetry with respect to a point iff it coincides with its rotation image through less than 360° about the point.

A figure is said to have n-fold rotation symmetry iff the smallest angle through which it can be turned to look exactly the same is $360^\circ/n$.



Def: A figure has reflection (line) symmetry with respect to a line iff it coincides with its reflection image through the line. The line is sometimes called the axis of symmetry.



Def: A pattern has translation symmetry iff it coincides with a translation image.

HW #1 (submitted Friday, 11/7) - Ch 1 Review Problems pp. 36-38

HW #2 (submitted Friday, 11/14) - Ch 2 Review Problems pp. 71-74, Ch 3 Review Problems pp. 124-128

HW #3 (submitted Friday, 11/21) - Ch 4 Review Problems pp.176-180

HW #4 (submitted Friday, 12/5) - Ch 5 Review Problems pp. 206-209

HW #5 (submitted Monday, 12/15) - Ch 6 Review Problems pp. 250-254

HW #6 (due Friday, 01/09)

- Ch 7 Review Problems pp. 292-295
- Khan Academy exercises: **anything that has been recommended by me!** ("Introduction to Euclidean geometry," "Angles and intersecting lines," "Congruence," etc.)
- Start working on Ch 8 Review Problems (pp. 325-329)

Test #3 - Friday, 01/16?