Postulate 8 - The Area Postulate

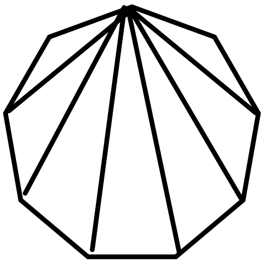
Every polygonal region has a positive number called its area such that

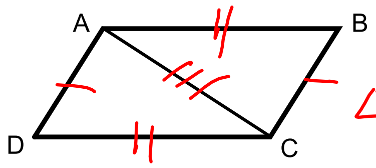
(1) congruent triangles have equal areas $\alpha\triangle ABC = \alpha\triangle CDA$

alpha means "area of"

(2) the area of a polygonal region is equal to the sum of the areas of its nonoverlapping parts

$$\alpha ABCD = \alpha\triangle ABC + \alpha\triangle CDA$$





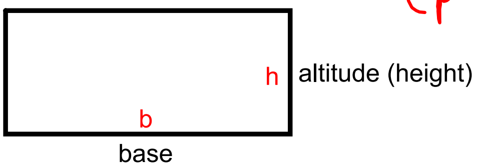
ABCD is a parallelogram

$\triangle ABC \cong \triangle CDA$

9.2 - Squares and Rectangles

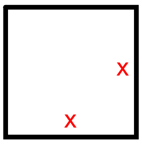
Postulate 9 - The area of a rectangle is the product of its base and altitude

(product of 2 perpendicular side lengths)



$A = bh$

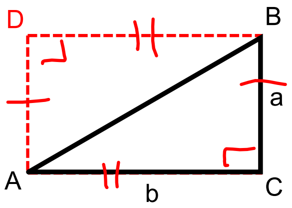
Corollary to Postulate 9 - The area of a square is the square of its side



$A = x^2$

9.3 - Triangles

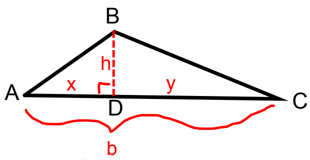
Theorem 38 - The area of a right triangle is half the product of its legs.



Given: Right $\triangle ABC$ with legs a and b

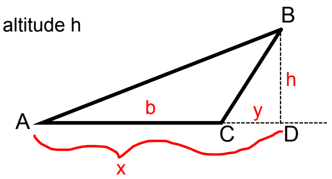
Prove: $\alpha \triangle ABC = \frac{1}{2}ba$

Theorem 39 - The area of a triangle is half the product of any base and corresponding altitude.



Given: $\triangle ABC$ with base b and altitude h

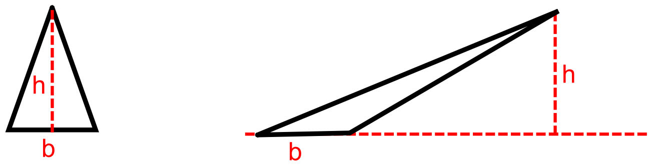
Prove: $\alpha \triangle ABC = \frac{1}{2}bh$



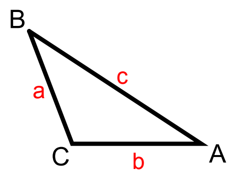
Proof

Statements	Reasons
1. $\triangle ABC$ has base b & altitude h	Given
2. $\triangle BDA$ & $\triangle BDC$ are right $\triangle$'s	Right $\triangle$'s have right angles
3. $\alpha\triangle BDA = \frac{1}{2}xh$, $\alpha\triangle BDC = \frac{1}{2}yh$	Area of a right $\triangle$ is $\frac{1}{2}$ base times altitude
4. $\alpha\triangle ABC = \alpha\triangle BDA + \alpha\triangle BDC$	Area Postulate
5. $\alpha\triangle ABC = \frac{1}{2}xh + \frac{1}{2}yh$	Substitution (#3 & 4)
6. $\alpha\triangle ABC = \frac{1}{2}h(x+y)$	Distributive
7. $x + y = b$ ($AD + DC = AC$)	Betweenness of Points Theorem
8. $\alpha\triangle ABC = \frac{1}{2}hb$	Substitution (#6 & 7)

Corollary to Theorem 39 - Triangles with equal bases and equal altitudes have equal areas.



Heron's Theorem



The area of a triangle with sides a, b, and c is $\sqrt{s(s-a)(s-b)(s-c)}$ where s is half of the triangle's perimeter.

$s = \frac{1}{2}(a+b+c)$

Suppose there are three triangles with the following sides:

Triangle 1: 5, 5, and 6. $s = \frac{1}{2}(5+5+6) = 8$ $A = \sqrt{\overset{8}{8}\cdot\overset{3}{(8-5)}\cdot\overset{3}{(8-5)}\cdot\overset{2}{(8-6)}} = 12$

Triangle 2: 5, 5, and 8. $s = \frac{1}{2}(5+5+8) = 9$ $A = \sqrt{\overset{9}{9}\cdot\overset{4}{(9-5)}\cdot\overset{4}{(9-5)}\cdot\overset{1}{(9-8)}} = 12$

Triangle 3: 5, 5, and 10. $s = \frac{1}{2}(5+5+10) = 10$ $A = \sqrt{\overset{10}{10}\cdot\overset{5}{(10-5)}\cdot\overset{5}{(10-5)}\cdot\overset{0}{(10-10)}} = 0$

Sum of 2 sides must be greater than 3rd side

1. Which triangle do you think has the greatest area?

2. Use Heron's Theorem to find the area of each triangle.

3. One of the "triangles" isn't really a triangle. Which one and why not?

△3 by Triangle Inequality

$\sqrt{9\cdot 16}$
$= \sqrt{9}\cdot\sqrt{16}$
$= 3\cdot 4$
$= 12$

Now, suppose there are two triangles with the following sides:

Triangle 4: 4, 6, and 8.

Triangle 5: 400, 600, and 1000.

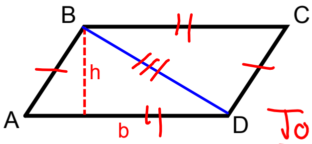
↑ not a △

4. Which do you think has the greater area?

5. ~~Use Heron's Theorem to find it.~~

9.4 - Parallelograms and Trapezoids

Theorem 40 - The area of a parallelogram is the product of any base and corresponding altitude.

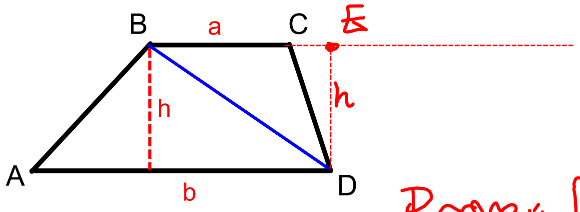


Given: ABCD is a parallelogram with base b and altitude h

To prove: $\alpha ABCD = bh$

Statements	Reasons
1. ABCD is a parallelogram w/ base b and altitude h	Given
2. Draw BD	2 points define a line
3. $AD = BC$ & $AB = CD$	Parallelograms have equal opposite sides
4. $BD = BD$	Reflexive
5. $\triangle ABD \cong \triangle CDB$	SSS congruence
6. $\alpha\triangle ABD = \alpha\triangle CDB$ $\alpha ABCD = \alpha\triangle ABD + \alpha\triangle CDB$	Area Postulate
7. $\alpha\triangle ABD = \frac{1}{2}bh$	Area of a triangle is $\frac{1}{2}$ base times altitude
8. $\alpha\triangle CDB = \frac{1}{2}bh$	Substitution (#6 & 7)
9. $\alpha ABCD = \frac{1}{2}bh + \frac{1}{2}bh$	Substitution (#6, 7, & 8)
10. $\alpha ABCD = bh$	Simplification

Theorem 41 - The area of a trapezoid is half the product of its altitude and the sum of its bases.



Given: trapezoid ABCD w/ bases a & b and altitude h

Prove: $\boxed{\alpha ABCD = \frac{1}{2}h(a+b)}$

$$\alpha ABCD = \alpha\triangle ABD + \alpha\triangle BCD$$

$$= \frac{1}{2}bh + \frac{1}{2}ah$$

$$\alpha ABCD = \frac{1}{2}h(a+b)$$

HW #7 (due Fri. 01/16)
- Ch 8 Review (pp. 325-329)
- Midterm Review (pp. 330-336)

Test #3 - Friday 01/16