

- Sign up for Khan Academy with coach code TDXMZZ
- Read Ch 1
- Ch 1 Review Problems pp. 36-38 #1-30 *all - due Friday*
- Read Ch 2
- Ch 2 Review Problems pp. 71-74 #1-19, 31-49

- Read Ch 3
- Ch 3 Review Problems pp. 124-128 #17-31, 34-49
- Ch 4 Review Problems pp. 176-180 #7-36, 48, 51, 52
- Ch 5 Review Problems pp. 206-209 #15-50
- Ch 6 Review Problems pp. 250-254 #9-19, 33-53

2.1 – Conditional Statements

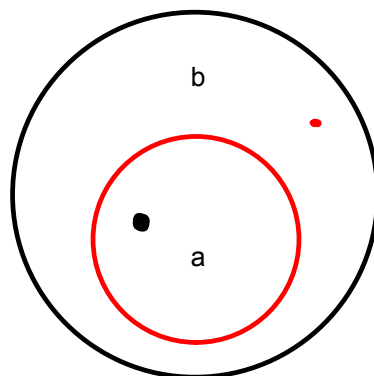
Conditional Statement: “If a, then b.” or “a implies b.”

a = hypothesis

b = conclusion

$$a \rightarrow b$$

Euler diagram



Rewrite the sentences in "if-then" form.

16. "Smokey Bear wouldn't have to do commercials for a living if money grew on trees."

If money grew on trees, then Smokey wouldn't have to go commercials.

17. "All architects use geometry."

If one is an architect, then one uses geometry.

19. "Use the stairs instead of the elevator in case of fire."

If there is a fire, then use the stairs instead of the elevator.

20. "No vampire casts a shadow."

If one is a vampire, then one does not cast a shadow.

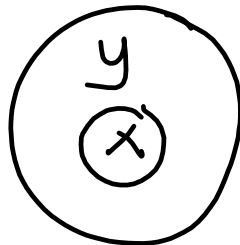
If one casts a shadow then one is not a vampire

$a \rightarrow b$
 $\sim b \rightarrow \sim a$ } logically equivalent
 contrapositives

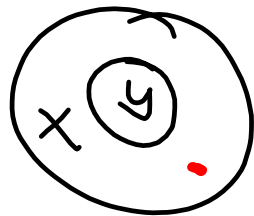
27. Draw Euler diagrams to represent the statements.

"If x, then y."

$x \rightarrow y$
 $\sim y \rightarrow \sim x$



"If y, then x."



29. Which diagram also illustrates the statement "If not y, then not x"?

2.2 – Definitions

When we define a word in mathematics, the word and its definition are understood to have the exact same meaning.

For example, if we define a “triangle” as “a polygon that has three sides,” then both of the following statements are true:

“If a figure is a triangle, then it is a polygon that has three sides.” $a \rightarrow b$ statement
 “If a figure is a polygon that has three sides, then it is a triangle.” $b \rightarrow a$ converse

For statement “If a, then b.”

Its converse is “If b, then a.”

In general, if a statement is true, then its converse is not necessarily true.

If a statement is a definition, then its converse is always true, and the two statements can be combined into an “if and only if” statement.

$$a \leftrightarrow b$$

“A figure is a triangle if and only if it is a polygon that has three sides.”

True statement: If you are an astronaut, you are not more than six feet tall.

Hypothesis:

$a =$ you are an astronaut

Conclusion:

$b =$ you are not more than six feet tall

6. Write the converse of the statement.

$b \rightarrow a$
 If you are not more than six feet tall,
 then you are an astronaut.

7. Is the converse true?

no

8. Does it have the same meaning as the original statement?

no

Definition: You have arachibutyrophobia iff you have the fear of peanutbutter sticking to the roof of your mouth.

If the definition were represented in symbols as $a \leftrightarrow b$ and "a" represents "arachibutyrophobia," what does

14. \leftrightarrow represent?

iff = if and only if

15. "b" represent?

the fear of peanut butter ... mouth

In words, " $a \rightarrow b$ " for this definition is "If you have arachibutyrophobia, then you are afraid of peanut butter sticking to the roof of your mouth."

16. Write in words, " $b \rightarrow a$ " for this definition.

If you are afraid of peanut butter...mouth, then you have arachibutyrophobia

17. Is this sentence necessarily true?

yes

(1) If it is your birthday, then you get some presents.

If birthday, then presents

$a \rightarrow b$ true

$b \nrightarrow a$

(2) Only if it is your birthday, do you get some presents.

If birthday, then presents

*$a \rightarrow b$ } true
 $b \rightarrow a$ }*

24. Is the first sentence true for you?

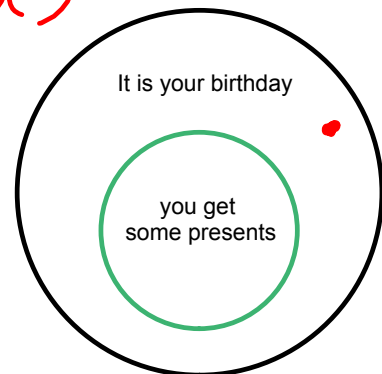


25. Is the second sentence true for you?

$b \rightarrow a$

27. Which sentence does this Euler Diagram illustrate?

neither



2.3 – Direct Proof

A syllogism is an argument of the form

$a \rightarrow b$

$b \rightarrow c$

Therefore, $a \rightarrow c$.

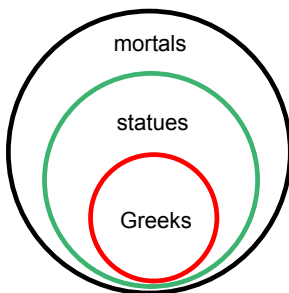
A syllogism is an example of a direct proof.

The statements $a \rightarrow b$ and $b \rightarrow c$ are called the premises of the argument.

$a \rightarrow c$ is called the conclusion of the argument, and is often considered to be a theorem.

A theorem is a statement that is proved by reasoning deductively from already accepted statements.

Syllogisms were discussed by the Greek philosopher Aristotle in the fourth century BC. Write the syllogism illustrated by the following Euler diagram.



*If one is Greek, then one is a statue.
 If one is a statue, then one is mortal,
 Hence, if one is Greek, then
 one is mortal.*

6. If the premises of a syllogism are true, does it follow that its conclusion must be true?

yes

7. If the premises of a syllogism are false, does it follow that its conclusion must be false?

no

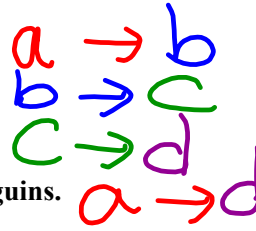
“Admit one ridiculous premise and the rest follows.” – Aristotle

If you live at the South Pole, you live in the Antarctic.

If you live in the Antarctic, you live where it is cold.

If you live where it is cold, you see a lot of penguins.

Therefore, if you live at the South Pole, you see a lot of penguins.



What part of the second premise matches

8. the conclusion of the first premise?

you live in the antarctic

9. the hypothesis of the third premise?

you live where it is cold

10. Starting with $a \rightarrow b$ to represent the first premise, represent the entire argument in symbols.

11. Which premise is ridiculous (false)?

$c \rightarrow d$

12. What does the fact that one premise is false indicate about the conclusion of the argument?

*can make no conclusion
maybe true. maybe false*