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2.3 – Direct Proof

A syllogism is an argument of the form

$a \rightarrow b$

$b \rightarrow c$

Therefore, $a \rightarrow c$.

A syllogism is an example of a direct proof.

The statements $a \rightarrow b$ and $b \rightarrow c$ are called the premises of the argument.

$a \rightarrow c$ is called the conclusion of the argument, and is often considered to be a theorem.

A theorem is a statement that is proved by reasoning deductively from already accepted statements.

If Captain Spaulding is in the jungle, there are too many cheetahs.

If there are too many cheetahs, Captain Spaulding can't play cards.

$$a \rightarrow b$$

$$b \rightarrow c$$

13. What conclusion follows from these premises?

If Captain Spaulding is in the jungle, then he can't play cards. $a \rightarrow c$

14. If the two premises are true, does it follow that the conclusion must be true?

yes

Write in the missing statements for the following proofs.

20. *Theorem:* If two hungry vultures took an airplane, they would be told that there is a limit of two carrion per passenger.

Proof:

If two hungry vultures took an airplane, they would want to take along some food.

> If the vultures want to take along some food, then they will try to carry on six dead raccoons.

If they tried to carry on six dead raccoons, the flight attendant would object.

> If the flight attendant objects then they will be told there is a limit of two carrion per passenger

$$a \rightarrow b$$

$$b \rightarrow c$$

$$c \rightarrow d$$

$$d \rightarrow e$$

Hence, the theorem

$$a \rightarrow e$$

21. *Theorem:* If a group of chess players checked into a hotel, the manager would say “I can’t stand chess nuts ~~boating~~ in an open foyer.”

Proof: **boasting**
 > If a group of chess players checks into a hotel, then they will stand in the lobby bragging about their tournament victories. $a \rightarrow b$

If they stood in the lobby bragging about their tournament victories, the manager would ask them to leave. $b \rightarrow c$

> If the manager asks them to leave, they would ask why. $c \rightarrow d$

If they asked why, the manager would say “I can’t stand chess nuts boasting in an open foyer.” $d \rightarrow e$

Here the Theorem

$a \rightarrow e$

2.4 – Indirect Proof

In an indirect proof, an assumption is made at the beginning that leads to a contradiction. The contradiction indicates that the assumption is false and the desired conclusion is true.

Direct versus Indirect proof of the theorem “If a, then d.”

Direct Proof:

- If a, then b.
- If b, then c.
- If c, then d.
- Therefore, if a, then d.

Indirect Proof:

- Suppose not d is true.
- If not d, then e.
- If e, then f,
- And so on until we come to a contradiction.
- Therefore, not d is false; so d is true.

List the assumption with which an indirect proof of each of the following statements would begin.

Example: If a tailor wants to make a coat last, he makes the pants first.

Answer: Suppose that he does not make the pants first.

4. If a teacher is cross-eyed, he has no control over his pupils.

Suppose a teacher has control over his pupils.

5. If a proof is indirect, then it leads to a contradiction.

Suppose a proof does not lead to a contradiction.

In a book written in the 13th century on the shape of the earth, the author reasoned: "If the earth were flat, the stars would rise at the same time for everyone, which they do not."

11. What is the author trying to prove?

the earth is not flat

12. With what assumption does the author begin?

the earth is flat

13. What is the contradiction?

the stars do not rise @ the same time for everyone

14. What does the contradiction prove about the author's beginning assumption?

it's false; hence the earth is not flat

Write the missing statements in the indirect proof:

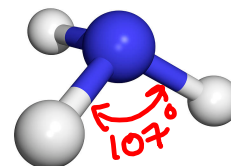
16. The ammonia molecule consists of three hydrogen atoms bonded to a nitrogen atom as shown in this figure.

The fact that chemists have found that each bond angle is 107° can be used to prove the following theorem.

Theorem: The atoms of an ammonia molecule are not coplanar.

Proof:

> *Suppose the atoms are coplanar.*



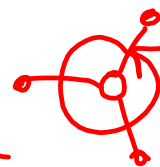
If the atoms are coplanar, then the sum of the three bond angles is 360° .

If the sum of the three bond angles is 360° , then each angle is 120° .

> *This contradicts the fact that the bond angles are each 107° .*

Therefore, our assumption is false and

> *The atoms are not coplanar.*



- ★ 19. A particular puzzle involves separating a set of twelve weights into two sets so that one set will exactly balance the other on a scale with two pans.

Consider this argument:

If a puzzle of this type has a solution, then the weights of the two sets will be equal.

If the weights of the two sets are equal, then each set will weigh half the total weight.

What conclusion follows from these two premises?

If a puzzle of this type has a solution, then each set of weights will weigh half the total weight.

20. Write in the missing statements in the indirect proof about this puzzle:

Theorem: If the sum of all of the weights is odd, then there is no solution.

Proof:

> Suppose there is a solution

If there is a solution, let the weights in one set add up to S .

If the weights in each set add up to S , then the weights in both sets add up to $S+S=2S$, an even number.

> This contradicts that the sum is odd

Therefore, our assumption is false and

> There is not a solution.

21. At a sports banquet there are 100 famous athletes. Each one is either a football player or a basketball player. At least one is a football player. Given any two of the athletes, at least one is a basketball player. **How many of the athletes are football players, and how many are basketball players? Construct an indirect argument to explain your reasoning.**

Theorem: There is only one football player and there are 99 basketball players.

Proof: Suppose there is more than one football player.