- Sign up for Khan Academy with coach code TDXMZZ
- Read Ch 1
- Ch 1 Review Problems pp. 36-38 #1-30 -- DUE FRIDAY 11/11
- Read Ch 2
- **Ch 2 Review Problems pp. 71-74 #1-19, 31-49 -- DUE THURSDAY 11/17**
- **Read Ch 3**
- Ch 3 Review Problems pp. 124-128 #17-31, 34-49 -- DUE WED 11/30
- Ch 4 Review Problems pp.176-180 #7-36, 48,51,52
- Ch 5 Review Problems pp. 206-209 #15-50
- Ch 6 Review Problems pp. 250-254 #9-19, 33-53

Quiz - today
Thursday - class will be held in B129
Friday - class is cancelled
Test #1 - Week after the break (Wed, Thurs, or Fri?)

2.3 – Direct Proof

A syllogism is an argument of the form

$a \rightarrow b$

$b \rightarrow c$

Therefore, $a \rightarrow c$.

A syllogism is an example of a direct proof.

The statements $a \rightarrow b$ and $b \rightarrow c$ are called the premises of the argument.

$a \rightarrow c$ is called the conclusion of the argument, and is often considered to be a theorem.

A theorem is a statement that is proved by reasoning deductively from already accepted statements.

2.4 – Indirect Proof

In an indirect proof, an assumption is made at the beginning that leads to a contradiction. The contradiction indicates that the assumption is false and the desired conclusion is true.

Direct versus Indirect proof of the theorem "If a, then d."

Direct Proof:

If a, then b.

If b, then c.

If c, then d.
Therefore, if a, then d.

Indirect Proof:

Suppose not d is true.

If not d, then e.

If e, then f,

And so on until we come to a contradiction.

Therefore, not d is false; so d is true.

2.5 – A Deductive System

To avoid circular definitions, mathematics leaves certain terms undefined.

Those which we have seen so far include: point, line, plane.

These undefined terms can be used to define other terms, for example,

Def: Points are collinear iff there is a line that contains all of them.

Def: Lines are concurrent iff they contain the same point.

Just as it is impossible to define everything without going around in circles, it is impossible to prove everything. We leave some statements unproved, and use them as a basis for building proofs of other statements.

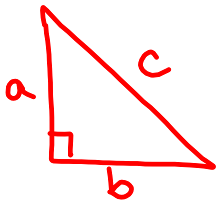
Def: A postulate is a statement that is assumed to be true without proof. Axiom/Postulate

Postulate 1: Two points determine a line.

Postulate 2: Three noncollinear points determine a plane.

2.6 - Some Famous Theorems of Geometry

The Pythagorean Theorem: The square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides.



a, b - legs
c - hypotenuse
$a^2 + b^2 = c^2$

The Triangle Sum Theorem: The sum of the angles in a triangle is 180°.

Circle Theorems:

If the diameter of a circle is d, then its circumference is πd. $C = \pi d = 2\pi r$

If the radius of a circle is r, then its area is πr^2.

3.1 – Number Operations and Equality

Algebraic Postulates of Equality:

Reflexive Property: a=a (Any number is equal to itself.)

Substitution Property: If a=b, then a can be substituted for b in any expression.

Addition Property: If a=b, then a+c=b+c

Subtraction Property: If a=b, then a-c=b-c.

Multiplication Property: If a=b, then ac=bc.

Division Property: If a=b, then a/c=b/c. , $c \neq 0$

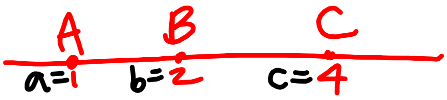
3.2 – The Ruler and Distance

Postulate 3: The Ruler Postulate – The points on a line can be numbered so that positive number differences measure distance.

Def: Betweenness of Points – A point is between two other points on the same line iff its coordinate is between their coordinates. (More briefly, A-B-C iff $a<b<c$ or $a>b>c$.)

Theorem 1: The Betweenness of Points Theorem: If A-B-C, then AB+BC=AC

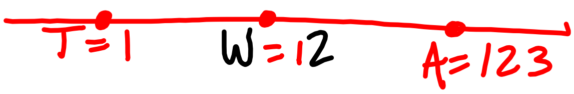
Proof for $a<b<c$ case:



Statements:	**Reasons:**
1. A-B-C	The hypothesis.
2. $a<b<c$ or $a>b>c$	Definition of betweenness.
3. AB=b-a and BC=c-b	Ruler Postulate.
4. AB+BC=(b-a)+(c-b)=c-a	Addition (and simplifaction).
5. AC=c-a	Ruler Postulate.
6. AB+BC=AC	Substitution (steps 4 and 5).

The distance between 2 real #'s a&b is $|b-a|$

Three points on a line have the following coordinates:
point A, 123; point T, 1; and point W, 12.

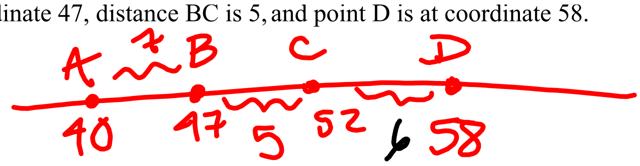


Which idea is the reason for each statement below (Ruler Postulate, definition of betweenness of points, or Betweenness of Points Theorem)?

4. T-W-A because $1<12<123$. def. of betweenness of points

5. TW+WA=TA because T-W-A. betweenness of points theorem

Suppose point A is at coordinate 40, point B is at coordinate 47, distance BC is 5, and point D is at coordinate 58. Determine:



1. The total distance AD.

58-40=18

2. The coordinate of C.

47+5=52

3. The distance CD.

58-52=6

Because A-B-C, AB+BC=AC, or 7+5=12, according to the Betweenness of Points Theorem. Use this theorem to complete the statements:

1. 9. Because B-C-D, BC+ CD = BD , or 5+ 6 = 11 .

2. 10. Because A-B-D, AB+ BD = AD , or 7+ 11 = 18 .

3. 11. Because A-C-D, AC+ CD = AD , or 12 + 6 = 18 .

Suppose AC=BD. Complete the statements:

38. Because A-B-C, AC= AB+BC (Betw. of pts. Thm)

39. Because B-C-D, BD= BC+CD

40. Why is AB+BC=BC+CD? Substitution

41. Why is AB=CD? subtraction (by BC on both sides)

3.3 – The Protractor and Angle Measure

Postulate 4: The Protractor Postulate – The rays in a half-rotation can be numbered from 0 to 180 so that positive number differences measure angles.

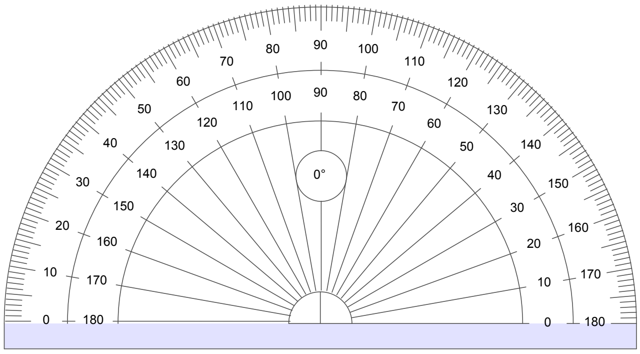
Definitions: An angle is

Acute iff it is less than 90°.

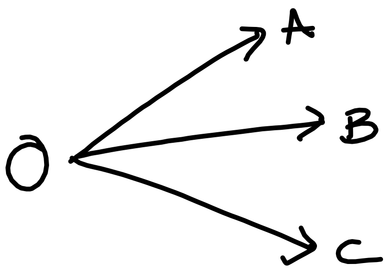
Right iff it is 90°.

Obtuse iff it is more than 90° but less than 180°.

Straight iff it is 180°.



Def: Betweenness of Rays – A ray is between two others in the same half-rotation iff its coordinate is between their coordinates. (More briefly, OA-OB-OC iff a<b<c or a>b>c.)



Theorem 2: The Betweenness of Rays Theorem –
If OA-OB-OC, then AOB+ BOC= AOC.

Proof for a>b>c case:

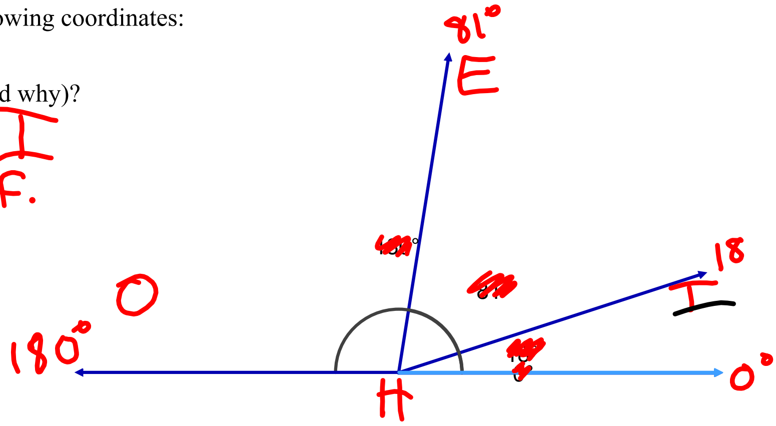
Statements:	**Reasons:**
1. OA-OB-OC	The hypothesis.
2. a>b>c	Definition of betweenness.
3. AOB=a-b and BOC=b-c	Protractor Postulate.
4. AOB+ BOC=(a-b)+(b-c)=a-c	Addition (and simplification).
5. AOC=a-c	Protractor Postulate.
6. AOB+ BOC= AOC	Substitution (steps 4 and 5).

Three rays in a half-rotation have the following coordinates:
ray HE, 81; ray HI, 18; and ray HO, 180.

4. Which ray is between the other two (and why)?

HO - HE - HI
Betw. of rays def.

Use your protractor to draw a figure.



5. Name and find the measures of the three angles formed by the rays.

∠OHE, ∠EHI, ∠OHI