Def: A parallelogram is a quadrilateral whose opposite sides are parallel.

Theorem 25: The opposite sides and angles of a parallelogram are equal.

Theorem 26: The diagonals of a parallelogram bisect each other.

Theorem 27: A quadrilateral is a parallelogram, if its opposite sides are equal.

Theorem 28: A quadrilateral is a parallelogram if its opposite angles are equal.

7.4 – Rectangles, Rhombuses, and Squares

Def: A square is a quadrilateral all of whose sides and angles are equal.

Every square is a rhombus.

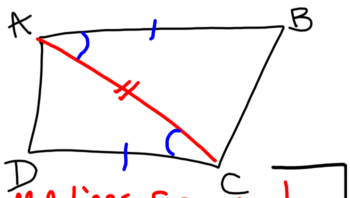
Def: A rhombus is a quadrilateral all of whose sides are equal.

Theorem 31: All rectangles are parallelograms.
Theorem 32: All rhombuses are parallelograms.

Theorem 29: A quadrilateral is a parallelogram if two opposite sides are both parallel and equal.

Given: ABCD is a quadrilateral
AB || DC & AB = CD

Prove: ABCD is a parallelogram

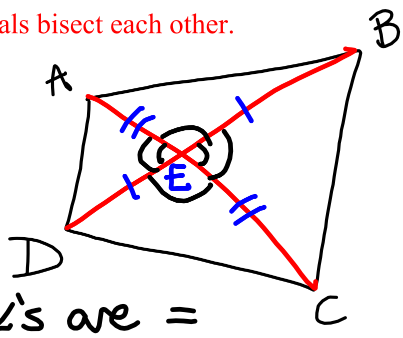


Proof :

1. $\angle A$ & $\angle D$ are supplementary, $\angle B$ & $\angle C$ are supplementary	parallel lines form suppl. interior $\angle$'s on same side of transversal
2. $\angle A + \angle D = 180°$, $\angle B + \angle C = 180°$	supplementary $\angle$'s sum to 180°
3. $\angle A + \angle D = \angle B + \angle C$	substitution
4. Draw AC	2 points define a line
5. AC = AC	reflexivity
6. $\angle BAC = \angle DCA$	parallel lines form equal alternate interior $\angle$'s
7. $\triangle BAC \cong \triangle DCA$	SAS congruence
8. AD = BC	corresponding parts of congruent $\triangle$'s are equal
9. ABCD is a parallelogram	quadrilateral w/ 2 pairs of equal opposite sides is a parallelogram

unnecessary

Theorem 30: A quadrilateral is a parallelogram if its diagonals bisect each other.



Given: ABCD w/ diagonals AC & BD that bisect each other

Prove: ABCD is a parallelogram

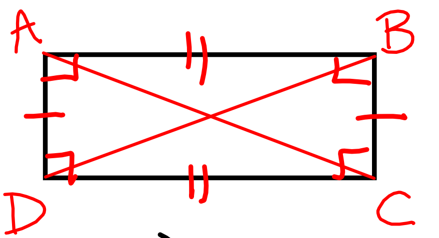
Proof:

1. ∠AEB = ∠DEC, ∠AED = ∠BEC — vertical ∠'s are =
2. AE = EC, DE = EB — bisector divides a segment into 2 equal segments
3. △AED ≅ △CEB, △AEB ≅ △CED — SAS congruences
4. AB = DC, AD = BC — corresponding parts of congruent △'s are =
5. ABCD is a parallelogram — a quadrilateral w/ 2 pairs of = opposite sides is a parallelogram

Theorem 33: The diagonals of a rectangle are equal.

Given: ABCD is a rectangle.

Prove: AC=BD.



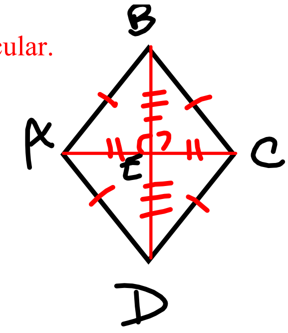
Proof:

1. ABCD is a parallelogram (all rectangles are parallelograms)
2. ∠A = ∠B = ∠C = ∠D — rectangles have all right ∠'s; all right ∠'s are equal
3. AB ∥ DC & AD ∥ BC — def. of parallelogram
4. AB = DC, AD = BC — parallelograms have = opposite sides
5. △ABC ≅ △BAD — SAS congruence
6. AC = BD — corresponding parts of congruent △'s are equal

Theorem 34: The diagonals of a rhombus are perpendicular.

Given: ABCD is a rhombus.

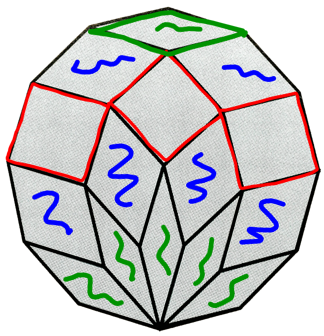
Prove: AC BD.



Proof :

1. AB = BC = CD = DA (def. of rhombus)
2. ABCD is a parallelogram (all rhombuses are parallelograms)
3. AC & BD bisect eachother — diagonals of a parallelogram bisect eachother
4. AE = EC & BE = ED — def. of bisector
5. $\triangle ABE \cong \triangle CBE$ — SSS congruence
6. $\angle AEB = \angle CEB$ — Corresponding parts of congruent $\triangle$'s are =
7. $AC \perp BD$ — If $\angle$'s in a linear pair are =, their sides are perpendicular

Regular dodecagon



1. How many sides does a dodecagon have?

12

A regular polygon is one that is equilateral and equiangular.

2. How many regular quadrilaterals do there seem to be in the figure?

3

3. What is a regular quadrilateral called?

squares

4. How many rectangles do there seem to be in the figure?

3 (the squares)

5. How many rhombuses are in the figure?

15

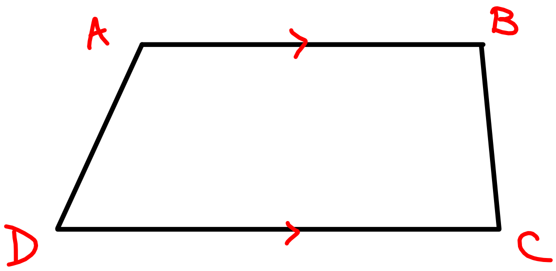
6. How many different shapes of rhombuses does the figure seem to contain?

3

7.5 – Trapezoids

Def: A trapezoid is a quadrilateral that has exactly one pair of parallel sides.

The parallel sides are called the bases of the trapezoid, and the non-parallel sides are called its legs. The pairs of angles that include each base are called base angles.



In this trapezoid:

Sides AB and DC are bases.

Sides AD and BC are legs.

Angles A and B are one pair of base angles.

Angles D and C are another pair of base angles.