Def: A parallelogram is a quadrilateral whose opposite sides are parallel.
Theorem 25: The opposite sides and angles of a parallelogram are equal.
Theorem 26: The diagonals of a parallelogram bisect each other.
Theorem 27: A quadrilateral is a parallelogram, if its opposite sides are equal.
Theorem 28: A quadrilateral is a parallelogram if its opposite angles are equal.
Theorem 29: A quadrilateral is a parallelogram if two opposite sides are both parallel and equal.
Theorem 30: A quadrilateral is a parallelogram if its diagonals bisect each other.

7.4 – Rectangles, Rhombuses, and Squares

Def: A square is a quadrilateral all of whose sides and angles are equal.
Every square is a rhombus.
Def: A rhombus is a quadrilateral all of whose sides are equal.
Theorem 31: All rectangles are parallelograms.
Theorem 32: All rhombuses are parallelograms.
Theorem 33: The diagonals of a rectangle are equal.
Theorem 34: The diagonals of a rhombus are perpendicular.

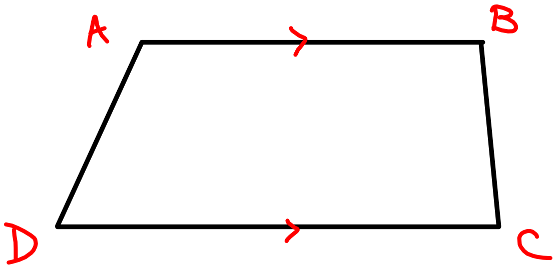
Quiz Next week on Trapezoids; Test #4 - Feb 6-8-ish?

HW due Mon. 01/30
proof of Thm 35
(base ∠'s of isoc. Δ are =)

7.5 – Trapezoids

Def: A trapezoid is a quadrilateral that has exactly one pair of parallel sides.

The parallel sides are called the bases of the trapezoid, and the non-parallel sides are called its legs. The pairs of angles that include each base are called base angles.



In this trapezoid:

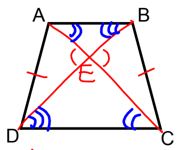
Sides AB and DC are bases.

Sides AD and BC are legs.

Angles A and B are one pair of base angles.

Angles D and C are another pair of base angles.

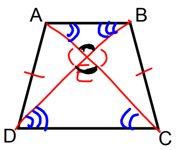
Def: An isosceles trapezoid is a trapezoid whose legs are equal.



Theorem 35: The base angles of an isosceles trapezoid are equal.
Given: ABCD is an isosceles trapezoid with bases AB and DC.
Prove: A= B and D= C

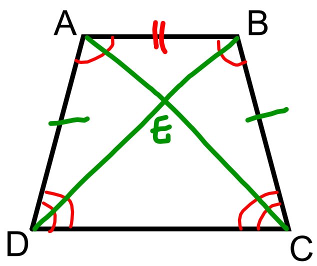
Proof:

1. AB ∥ DC — base sides of a trapezoid are parallel
2. ∠A & ∠D are supplementary, ∠B & ∠C are supplementary — parallel lines form supplementary interior ∠'s on the same side of the transversal
3. ∠A+∠D = 180°, ∠B+∠C = 180° — Supplementary ∠'s sum to 180°
4. ∠A+∠D = ∠B+∠C — substitution
5. Draw BD & AC — 2 points define a line
6. ∠AED = ∠BEC, ∠AEB = ∠DEC — Vertical ∠'s are =
7. ∠BAC = ∠DCA & ∠ABD = ∠CDB — parallel lines form = alternate interior ∠'s
8. ∠DAC+∠CAB = ∠A, ∠ABD+∠DBC = ∠B, ∠ADB+∠BDC = ∠D, ∠DCA+∠ACB = ∠C — Betw. of Rays Thm



9. ∠DAC+∠CAB+∠ADB+∠BDC = ∠ABD+∠DBC+∠DCA+∠ACB — Subst. A+D = B+C
10. ∠DAC+∠ADB = ∠DBC+∠ACB — subst. & subtr.
11. ∠BEC = ∠CAB+∠ABD = ∠BDC+∠ACD, ∠AEB = ∠DBC+∠ACB = ∠DAC+∠ADB — Exterior ∠ Theorem

Theorem 36 : The diagonals of an isosceles trapezoid are equal.
Given: ABCD is an isosceles trapezoid with bases AB and DC.
Prove: DB=CA.

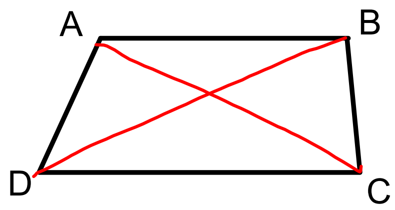


Proof :

1. ∠A = ∠B , ∠D = ∠C — base ∠'s of isoc. are =
2. Draw AC & BD — 2pts define a line
3. AD = BC — legs of isoc. trap. are =
4. ~~∠CAB = ∠DCA, ∠ABD = ∠BDC~~ — ~~parallel lines form = alt. int ∠'s~~
5. ~~vertical ∠'s~~ — AB = AB
6. △DAB ≅ △CBA — SAS congruence
7. DB = CA — corresponding parts of ≅ △'s are =

If a quadrilateral is a trapezoid, then its diagonals cannot bisect each other.

Given: ABCD is a trapezoid
Prove: AC and DB do not bisect each other.



Proof

1. Suppose AC & BD do bisect each other
2. ABCD is a parallelogram

BUT this contradicts that ABCD is a trapezoid. Here, our assumption is false and AC & BD cannot bisect each other