

2.3 – Direct Proof

A syllogism is an argument of the form

$a \rightarrow b$

$b \rightarrow c$

Therefore, $a \rightarrow c$.

A syllogism is an example of a direct proof.

The statements $a \rightarrow b$ and $b \rightarrow c$ are called the premises of the argument.

$a \rightarrow c$ is called the conclusion of the argument, and is often considered to be a theorem.

A theorem is a statement that is proved by reasoning deductively from already accepted statements.

2.4 – Indirect Proof

In an indirect proof, an assumption is made at the beginning that leads to a contradiction. The contradiction indicates that the assumption is false and the desired conclusion is true.

Direct versus Indirect proof of the theorem “If a, then d.”

Direct Proof:

If a, then b.

If b, then c.

If c, then d.

Therefore, if a, then d.

Indirect Proof:

Suppose not d is true.

If not d, then e.

If e, then f,

And so on until we come to a contradiction.

Therefore, not d is false; so d is true.

List the assumption with which an indirect proof of each of the following statements would begin.

Example: If a tailor wants to make a coat last, he makes the pants first.

Answer: Suppose that he does not make the pants first.

4. If a teacher is cross-eyed, he has no control over his pupils.

Suppose he does have control over his pupils

5. If a proof is indirect, then it leads to a contradiction.

Suppose it does not lead to a contradiction

In a book written in the 13th century on the shape of the earth, the author reasoned: "If the earth were flat, the stars would rise at the same time for everyone, which they do not."

11. What is the author trying to prove?

The earth is not flat

12. With what assumption does the author begin?

The earth is flat

13. What is the contradiction?

the stars don't rise @ the same time

14. What does the contradiction prove about the author's beginning assumption?

initial assumption is false, hence the earth is not flat.