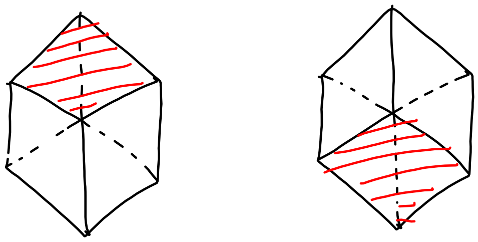


2.3 – Direct Proof

A syllogism is an argument of the form

a→b

b→c

Therefore, a→c.

Theorem: If a, then d.
If a, then b.
If b, then c.
If c, then d.
Therefore, if a, then d.

A syllogism is an example of a direct proof.

The statements a→b and b→c are called the premises of the argument.

a→c is called the conclusion of the argument, and is often considered to be a theorem.

A theorem is a statement that is proved by reasoning deductively from already accepted statements.

2.4 – Indirect Proof

In an indirect proof, an assumption is made at the beginning that leads to a contradiction. The contradiction indicates that the assumption is false and the desired conclusion is true.

Direct versus Indirect proof of the theorem "If a, then d."

Direct Proof:

If a, then b.

If b, then c.

If c, then d.
Therefore, if a, then d.

Indirect Proof:

Suppose not d is true.

If not d, then e.

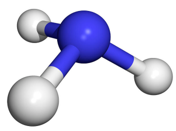
If e, then f,

And so on until we come to a contradiction.

Therefore, not d is false; so d is true.

Write the missing statements in the indirect proof:

16. The ammonia molecule consists of three hydrogen atoms bonded to a nitrogen atom as shown in this figure.

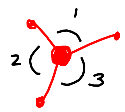


The fact that chemists have found that each bond angle is 107° can be used to prove the following theorem.

Theorem: The atoms of an ammonia molecule are not coplanar.

Proof:

> Assume the atoms are coplanar.



If the atoms are coplanar, then the sum of the three bond angles is 360°.

If the sum of the three bond angles is 360°, then each angle is 120°.

> This contradicts the fact that each bond angle is 107°.

Therefore, our assumption is false and

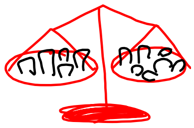
> hence, the atoms of an ammonia molecule are not coplanar.

19. A particular puzzle involves separating a set of twelve weights into two sets so that one set will exactly balance the other on a scale with two pans.

Consider this argument:

If a puzzle of this type has a solution, then the weights of the two sets will be equal.

If the weights of the two sets are equal, then each set will weigh half the total weight.



What conclusion follows from these two premises?

$\left\{\begin{array}{l}\text{If } a, \text{ then } b \\ \text{If } b, \text{ then } c\end{array}\right\} \Longrightarrow$ If a, then c.

If a puzzle of this type has a solution, then each set will weigh half the total weight.

20. **Write in the missing statements in the indirect proof about this puzzle:**

Theorem: If $\underbrace{\text{the sum of all of the weights is odd}}_{a}$, then $\underbrace{\text{there is no solution}}_{b}$.

Proof:

> Assume (not b) there is a solution.

If there is a solution, let the weights in one set add up to S.

If the weights in each set add up to S, then the weights in both sets add up to S+S=2S, an even number.

> This contradicts that the sum of all the weights is odd.

Therefore, our assumption is false and

> Hence, there is no solution.

21. At a sports banquet there are 100 famous athletes. Each one is either a football player or a basketball player. At least one is a football player. Given any two of the athletes, at least one is a basketball player. **How many of the athletes are football players, and how many are basketball players? Construct an indirect argument to explain your reasoning.**

Theorem: There is 1 football player and there are 99 basketball players.

Proof: Suppose there is more than one football player. If there is more than one football player, then we could have a group of two football players. This contradicts that any pair of athletes must contain at least one basketball player. Therefore, our assumption is false. Hence only one of the 100 athletes is a football player.

assumption

2.5 – A Deductive System

To avoid circular definitions, mathematics leaves certain terms undefined.

Those which we have seen so far include: point, line, plane.

These undefined terms can be used to define other terms, for example,

Def: Points are collinear iff there is a line that contains all of them.

Def: Lines are concurrent iff they contain the same point.

(intersect)

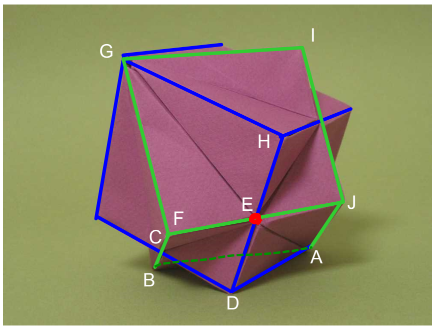
Just as it is impossible to define everything without going around in circles, it is impossible to prove everything. We leave some statements unproved, and use them as a basis for building proofs of other statements.

Def: A postulate is a statement that is assumed to be true without proof.

axiom = postulate

Postulate 1: Two points determine a line.

Postulate 2: Three noncollinear points determine a plane.



Determine if the following statements are true or false:

19. Points B, C, and F are collinear. T

20. Points B and C determine a line. T

21. Points F, E, and J are coplanar. T

22. Points F, E, and J determine a plane. F
because they are collinear

23. Points A, E, and G are collinear. F

24. Points A, B, C, and J are coplanar. T

25. Lines DH, FJ, and EG are concurrent. T
@ point E.