2.3 – Direct Proof

A syllogism is an argument of the form

Theorem: If a, then d.
If a, then b.
If b, then c.
If c, then d.
Therefore, if a, then d.

$a \rightarrow b$

$b \rightarrow c$

Therefore, $a \rightarrow c$.

A syllogism is an example of a direct proof.

The statements $a \rightarrow b$ and $b \rightarrow c$ are called the premises of the argument.

$a \rightarrow c$ is called the conclusion of the argument, and is often considered to be a theorem.

A theorem is a statement that is proved by reasoning deductively from already accepted statements.

2.4 – Indirect Proof

In an indirect proof, an assumption is made at the beginning that leads to a contradiction. The contradiction indicates that the assumption is false and the desired conclusion is true.

Direct versus Indirect proof of the theorem "If a, then d."

Direct Proof:

If a, then b.

If b, then c.

If c, then d.
Therefore, if a, then d.

Indirect Proof:

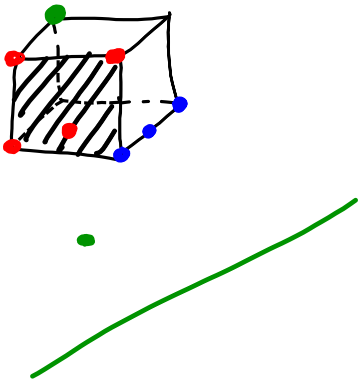
Suppose not d is true.

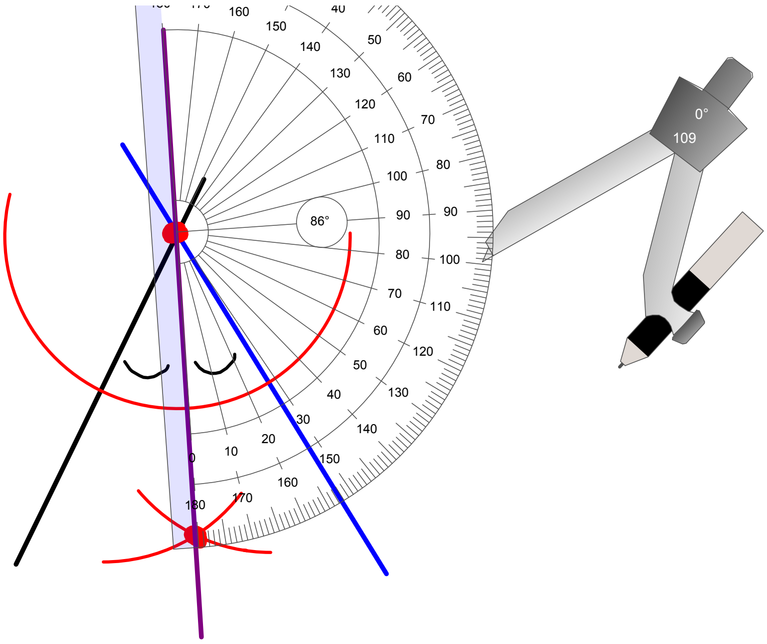
If not d, then e.

If e, then f,

And so on until we come to a contradiction.

Therefore, not d is false; so d is true.




86°
0°
109

2.5 – A Deductive System

To avoid circular definitions, mathematics leaves certain terms undefined.

Those which we have seen so far include: point, line, plane.

These undefined terms can be used to define other terms, for example,

Def: Points are collinear iff there is a line that contains all of them.

Def: Lines are concurrent iff they contain the same point.

(intersect)

Just as it is impossible to define everything without going around in circles, it is impossible to prove everything. We leave some statements unproved, and use them as a basis for building proofs of other statements.

Def: A postulate is a statement that is assumed to be true without proof.

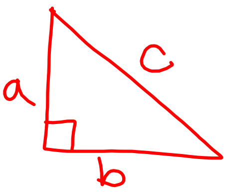
axiom = postulate

Postulate 1: Two points determine a line.

Postulate 2: Three noncollinear points determine a plane.

2.6 - Some Famous Theorems of Geometry

The Pythagorean Theorem: The square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides.



$a^2 + b^2 = c^2$

If ABC is a right triangle with legs of length a and b and hypotenuse of length c, then $a^2 + b^2 = c^2$.

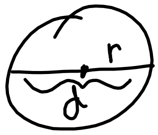
The Triangle Sum Theorem: The sum of the angles in a triangle is 180°.

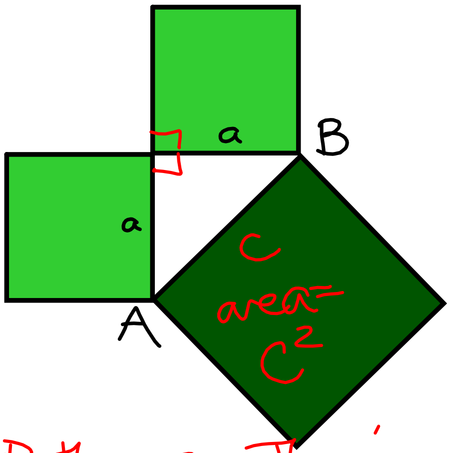
If ABC is a triangle, then

Circle Theorems: $\angle 1 + \angle 2 + \angle 3 = 180°$

If the diameter of a circle is d, then its circumference is πd.

If the radius of a circle is r, then its area is πr^2.





Pythagorean Thm:
$a^2+a^2=c^2$

34. The area of the light green square is

a^2

35. The combined area of the two light green squares is

$a^2+a^2=2a^2$

36. The area of the dark green square is

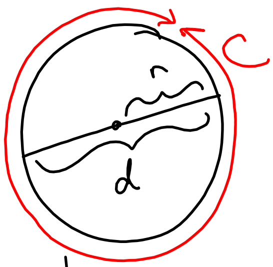
$2a^2$

41. "The area of a circle is half of the circumference multiplied by half of the diameter."
~ 6th century Indian astronomer Aryabhata

Is this true? yes.

$C=\pi d$; $d=2r$
$A=\pi r^2$

If r, d, C, and A are the radius, diameter, Circumference, and area, respectively, of a circle, then $A=\left(\frac{1}{2}C\right)\left(\frac{1}{2}d\right)$.



Proof:

If C is the circumference, then $C=\pi d$.
If d is the diameter, then $d=2r$.
If $C=\pi d$ and $d=2r$, then $\left(\frac{1}{2}C\right)\left(\frac{1}{2}d\right)=\left(\frac{1}{2}\cdot\pi\cdot 2r\right)\left(\frac{1}{2}\cdot 2r\right)$
$\left(\cancel{\frac{1}{2}}\pi\cdot\cancel{2}r\right)\left(\cancel{\frac{1}{2}}\cdot\cancel{2}r\right)$ simplifies to πr^2, and $A=\pi r^2$.
Therefore $A=\left(\frac{1}{2}C\right)\left(\frac{1}{2}d\right)$.