3.1 – Number Operations and Equality

Algebraic Postulates of Equality:

Reflexive Property: a=a (Any number is equal to itself.)

Substitution Property: If a=b, then a can be substituted for b in any expression.

Addition Property: If a=b, then a+c=b+c

Subtraction Property: If a=b, then a-c=b-c.

Multiplication Property: If a=b, then ac=bc.

Division Property: If a=b, then a/c=b/c. , $c \neq 0$

State the property of equality illustrated by each statement:

3. If c/d=π, then c=πd

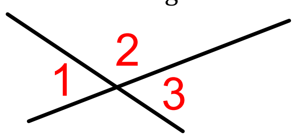
$\cancel{d} \cdot \frac{c}{\cancel{d}} = \pi \cdot d$ multiplication property of equality

4. If $\underbrace{\angle A+\angle B}_{\angle C}+\angle C=180°$ and ∠C=∠A+∠B, then ∠C+∠C=180°.

substitution prop. of eq.

5. If 2∠C=180°, then ∠C=90°. division prop. of eq.

This figure shows two lines intersecting to form several angles, three of which are numbered.



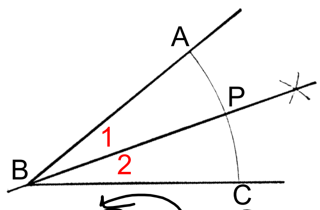
8. If ∠1+∠2=∠2+∠3, then ∠1=∠3. Why?

subtraction (by ∠2)

9. If ∠1=∠2 and ∠2=∠3, then ∠1=∠3. Why?

Substitution

This figure shows how we bisected an angle by using a straightedge and compass. Let's check the algebra to see that ∠1 is the size that we would expect.



11. If ∠ABC=∠1+∠2 and ∠1=∠2, then ∠ABC=∠1+∠1=2∠1. Why?

substitution simplification

12. If ∠ABC=2∠1, then ∠1=(1/2) ∠ABC. Why?

division

Quadratic formula

$\boxed{\text{If } ax^2 + bx + c = 0, \text{ then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.}$

37. What is the hypothesis of this theorem?

$ax^2+bx+c=0$

Name the postulate that is the reason for each of the following first three steps in its proof:

38. If $ax^2 + bx + c = 0$, then $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$.

division (by a)

39. If $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$, then $x^2 + \frac{b}{a}x = -\frac{c}{a}$.

subtraction (by $\frac{c}{a}$)

40. If $x^2 + \frac{b}{a}x = -\frac{c}{a}$, then $x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$.

addition by $\left(\frac{b}{2a}\right)^2$

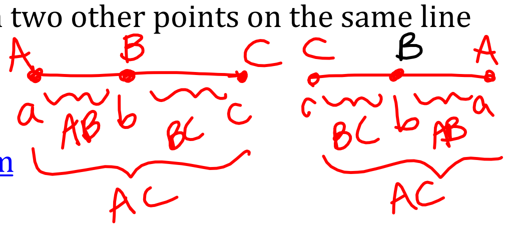
41. What kind of proof begins like this?

direct proof

3.2 – The Ruler and Distance

Postulate 3: The Ruler Postulate – The points on a line can be numbered so that positive number differences measure distance.

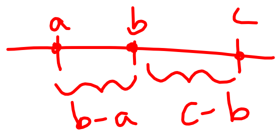
Def: Betweenness of Points – A point is between two other points on the same line iff its coordinate is between their coordinates. (More briefly, A-B-C iff a<b<c or a>b>c.)



Theorem 1: The Betweenness of Points Theorem

$\boxed{\text{If A-B-C, then AB+BC=AC}}$

Proof for a<b<c case:



Statements:	**Reasons:**
1. A-B-C	The hypothesis.
2. a<b<c	Definition of betweenness.
3. AB=b-a and BC=c-b	Ruler Postulate.
4. AB+BC=(b-a)+(c-b)=c-a	& substitution Addition (and simplifaction).
5. AC=c-a	Ruler Postulate.
6. AB+BC=AC	Substitution (steps 4 and 5).