

3.1 - Number Operations and Equality

Algebraic Postulates of Equality:

Reflexive Property: $a=a$ (Any number is equal to itself.)

Substitution Property: If $a=b$, then a can be substituted for b in any expression.

Addition Property: If $a=b$, then $a+c=b+c$

Subtraction Property: If $a=b$, then $a-c=b-c$.

Multiplication Property: If $a=b$, then $ac=bc$.

Division Property: If $a=b$, then $a/c=b/c$. $c \neq 0$

3.2 - The Ruler and Distance

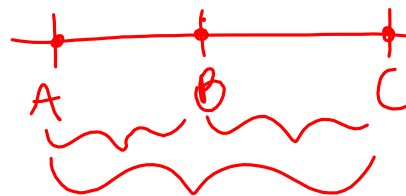
Postulate 3: The Ruler Postulate – The points on a line can be numbered so that positive number differences measure distance.

Def: **Betweenness of Points** – A point is between two other points on the same line iff its coordinate is between their coordinates.

(More briefly, $A-B-C$ iff $a < b < c$ or $a > b > c$.)

Theorem 1: The Betweenness of Points Theorem

If $A-B-C$, then $AB+BC=AC$



Three points on a line have the following coordinates:

point A, 123; point T, 1; and point W, 12.



Which idea is the reason for each statement below (Ruler Postulate, definition of betweenness of points, or Betweenness of Points Theorem)?

4. T-W-A because $1 < 12 < 123$.

def'n of betweenness

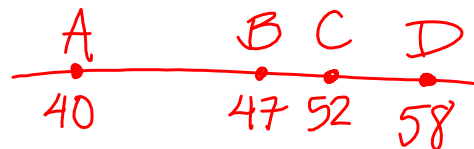
5. $TW + WA = TA$ because T-W-A.

Betweenness of Points Theorem

Suppose point A is at coordinate 40, point B is at coordinate 47, distance BC is 5, and point D is at coordinate 58. Determine:

1. The total distance AD.

$58 - 40 = \boxed{18}$ (Ruler Postulate)



2. The coordinate of C.

$47 + 5 = \boxed{52}$ $52 - 47 = 5$

3. The distance CD.

$58 - 52 = \boxed{6}$

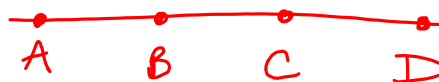
Because A-B-C, $AB + BC = AC$, or $7 + 5 = 12$, according to the Betweenness of Points Theorem. Use this theorem to complete the statements:

1. 9. Because B-C-D, $BC + \text{CD} = \text{BD}$, or $5 + \boxed{6} = \boxed{11}$.

2. 10. Because A-B-D, $AB + \text{BD} = \text{AD}$, or $7 + \boxed{11} = \boxed{18}$.

3. 11. Because A-C-D, $AC + \text{CD} = \text{AD}$, or $\boxed{12} + \boxed{6} = \boxed{18}$.

Suppose $AC=BD$. Complete the statements:



38. Because A-B-C, $AC= AB+BC$
 (Betweenness of Points Theorem)

39. Because B-C-D, $BD= BC+CD$

40. Why is $AB+BC=BC+CD$? *substitution property of equality*

41. Why is $AB=CD$? *subtraction property of equality*

3.3 - The Protractor and Angle Measure

Postulate 4: The Protractor Postulate – The rays in a half-rotation can be numbered from 0 to 180 so that positive number differences measure angles.

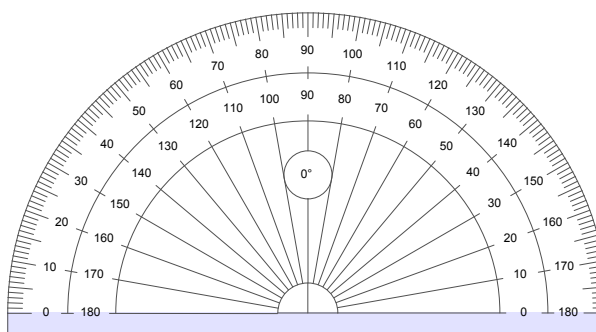
Definitions: An angle is

Acute iff it is less than 90° .

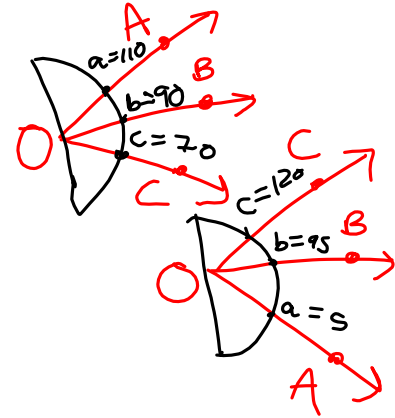
Right iff it is 90° .

Obtuse iff it is more than 90° but less than 180° .

Straight iff it is 180° .



Def: **Betweenness of Rays** – A ray is between two others in the same half-rotation iff its coordinate is between their coordinates.
 (More briefly, $OA-OB-OC$ iff $a < b < c$ or $a > b > c$.)



Theorem 2: The Betweenness of Rays Theorem –

If $OA-OB-OC$, then $\angle AOB + \angle BOC = \angle AOC$.

Proof for $a > b > c$ case:

Statements:

1. $OA-OB-OC$
2. $a > b > c$ (or $a < b < c$)
3. $\angle AOB = a - b$ and $\angle BOC = b - c$
4. $\angle AOB + \angle BOC = (a - b) + (b - c) = a - c$
5. $\angle AOC = a - c$
6. $\angle AOB + \angle BOC = \angle AOC$

Reasons:

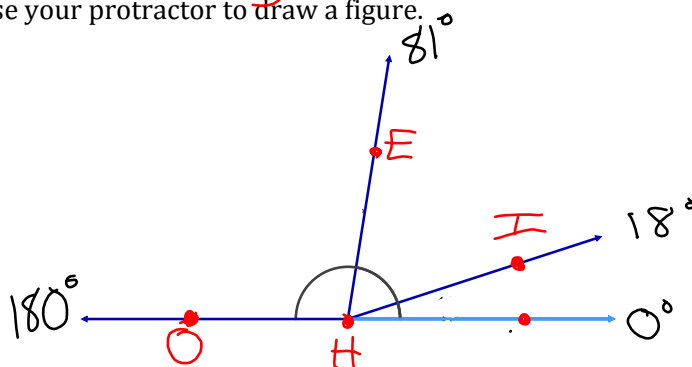
Given
 Betweenness of Points Theorem
 Protractor Postulate
 Addition, Substitution, & Simplification
 Protractor Postulate
 Substitution (#4 & 5)

Three rays in a half-rotation have the following coordinates:
 ray HE, 81; ray HI, 18; and ray HO, 180.

4. Which ray is between the other two (and why)?

$HI-HE-HO$ def. of betweenness of rays
 (18 < 81 < 180)

Use your protractor to draw a figure.



5. Name and find the measures of the three angles formed by the rays.

$$\angle IHE = 81 - 18 = 63$$

$$\angle EHO = 180 - 81 = 99$$

$$\angle IHO = 180 - 18 = 63 + 99 = 162$$