3.1 – Number Operations and Equality

Algebraic Postulates of Equality:

Reflexive Property: a=a (Any number is equal to itself.)

Substitution Property: If a=b, then a can be substituted for b in any expression.

Addition Property: If a=b, then a+c=b+c

Subtraction Property: If a=b, then a-c=b-c.

Multiplication Property: If a=b, then ac=bc.

Division Property: If a=b, then a/c=b/c.) $c \neq 0$

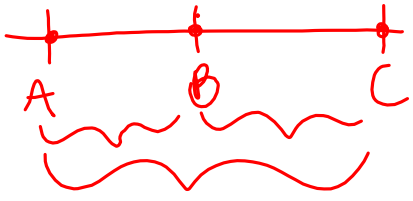
3.2 – The Ruler and Distance

Postulate 3: The Ruler Postulate – The points on a line can be numbered so that positive number differences measure distance.

Def: Betweenness of Points – A point is between two other points on the same line iff its coordinate is between their coordinates.
(More briefly, A-B-C iff a<b<c or a>b>c.)

Theorem 1: The Betweenness of Points Theorem

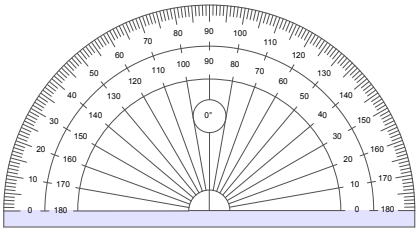
If A-B-C, then AB+BC=AC



3.3 – The Protractor and Angle Measure

Postulate 4: The Protractor Postulate – The rays in a half-rotation can be numbered from 0 to 180 so that positive number differences measure angles.

Definitions: An angle is



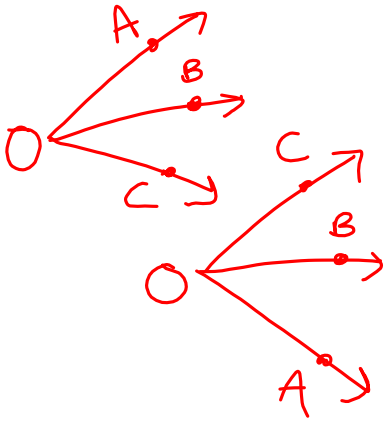
Acute iff it is less than 90°.

Right iff it is 90°.

Obtuse iff it is more than 90° but less than 180°.

Straight iff it is 180°.

Def: Betweenness of Rays – A ray is between two others in the same half-rotation iff its coordinate is between their coordinates.
(More briefly, OA-OB-OC iff a<b<c or a>b>c.)



Theorem 2: The Betweenness of Rays Theorem – If OA-OB-OC, then ∠AOB+∠BOC=∠AOC.

3.4 – Bisection

Def: A point is on the midpoint of a line segment iff it divides the line segment into two equal segments.

Def: A line bisects an angle iff it divides the angle into two equal angles.

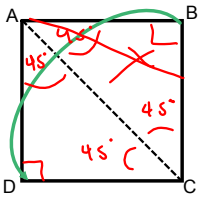
Def: Two objects are congruent if and only if they coincide exactly when superimposed.

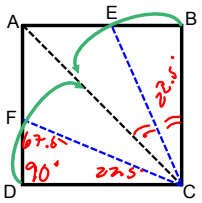
Def: A corollary is a theorem that can be easily proved as a consequence of a postulate or another theorem.

Corollary to the Ruler Postulate: A line segment has exactly one midpoint.

Corollary to the Protractor Postulate: An angle has exactly one ray that bisects it.

Bisecting angles with origami: Starting with a square sheet of paper, corner B is folded onto D. Then sides BC and DC are folded onto the fold AC.





Because ∠BAC fits onto ∠DAC, ∠BAC and ∠DAC are congruent.

17. Which angle is bisected if ∠BAC=∠DAC?

(∠A)=∠DAB=∠BAD

18. Name three more angles that are bisected in the folding process.

∠ACB (∠BCA), ∠BCD, ∠DCA (∠ACD)

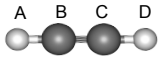
Angle BCD is a right angle because the process starts with a square. Find the number of degrees in each of the following angles.

20. ∠FCD 22.5°

21. ∠FCE 45°

23. ∠DFC 67.5°

Acetylene molecules contain four atoms, arranged linearly.



34. In this molecule, AB=CD, A-B-C and B-C-D. Use these facts to supply the reasons in the following direct proof that AC=BD.

Proof:

Statements	**Reasons**
AB=CD	Given
AB+BC=BC+CD	Addition Property of Equality
A-B-C and B-C-D	Given
AB+BC=AC and BC+CD=BD	Betweenness of Points Theorem
Therefore, AC=BD	Substitution (#2&4)

35. Use the additional fact that AC>2AB to supply the missing statements and reasons in this indirect proof that B is *not* the midpoint of AC.

Proof:

Statements	**Reasons**
B is the midpoint of AC	Assumption
If B is the midpoint of AC, then AB=BC.	definition of midpoint
Because AB+BC=AC, 2AB=AC.	Betw. of Pts. Thm & Substitution
AC > 2AB	Hypothesis
Therefore, our assumption is false and B is not the midpoint of AC.	Contradiction

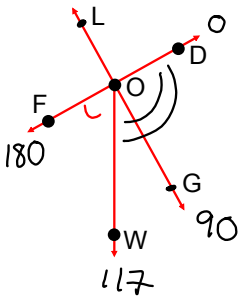
3.5 – Complementary and Supplementary Angles

Def: Two angles are complementary iff their sum is 90°.

Def: Two angles are supplementary iff their sum is 180°.

Theorem 3: Complements of the same angle are equal. (proved on p.106)

Theorem 4: Supplements of the same angle are equal.



If a protractor is placed on the figure so that OD has coordinate 0, the coordinates of the other rays are: OG, 90; OW, 117; OF, 180.

16. Write the equation that follows from the fact that OD-OW-OF.

$\angle FOW + \angle WOD = \angle FOD$

17. Find the measures of ∠DOW 117°

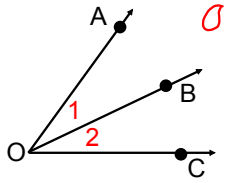
∠WOF 63°

∠DOF 180°

18. What relation does ∠DOW have to ∠WOF? supplementary

19. Find the measure of ∠WOG $117 - 90 = 27°$

20. What relation does ∠WOG have to ∠WOF? complementary



$OA - OB - OC$

$\Rightarrow \angle AOB + \angle BOC = \angle AOC$

$\angle 1 + \angle 2 = \angle AOC$

In the figure, ∠1 and ∠2 are both complements of ∠AOC.

44. What else is true? $\overrightarrow{OB}$ bisects ∠AOC

$\angle 1 = \angle 2$; $\angle 1 + \angle AOC = 90°$, $\angle 2 + \angle AOC = 90°$

45. Is it possible to figure out the size of each angle in the figure without measuring them?