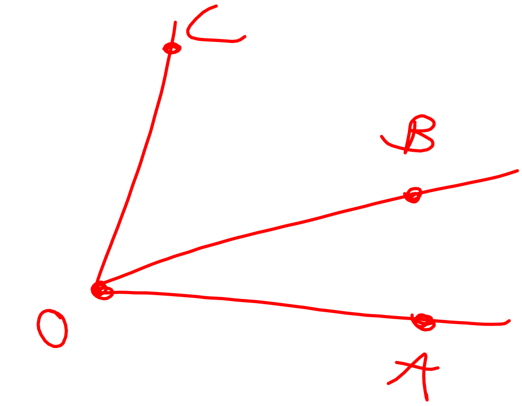
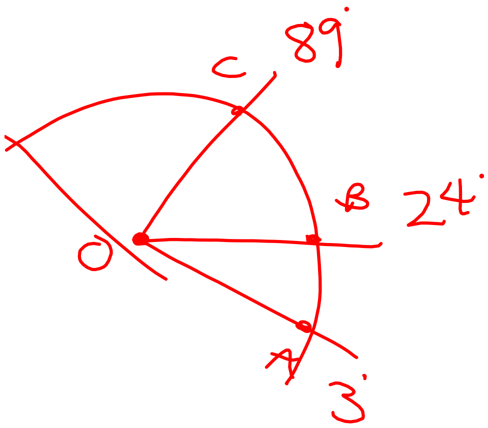


$\angle AOC = 80^\circ$

$\angle AOB = 20^\circ$

$\angle BOC = 60^\circ$

$\angle AOB + \angle BOC = \angle AOC$



$\angle AOB = 24 - 3$

$\angle BOC = 89 - 24$

$\angle AOC = 89 - 3$

3.1 – Number Operations and Equality

Algebraic Postulates of Equality:

Reflexive Property: a=a (Any number is equal to itself.)

Substitution Property: If a=b, then a can be substituted for b in any expression.

Addition Property: If a=b, then a+c=b+c

Subtraction Property: If a=b, then a-c=b-c.

Multiplication Property: If a=b, then ac=bc.

Division Property: If a=b, then a/c=b/c. , $c \neq 0$

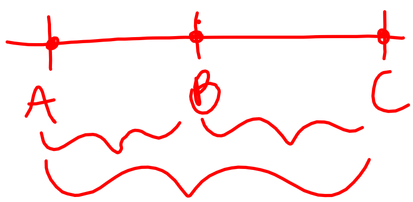
3.2 – The Ruler and Distance

Postulate 3: The Ruler Postulate – The points on a line can be numbered so that positive number differences measure distance.

Def: Betweenness of Points – A point is between two other points on the same line iff its coordinate is between their coordinates.
(More briefly, A-B-C iff a<b<c or a>b>c.)

Theorem 1: The Betweenness of Points Theorem

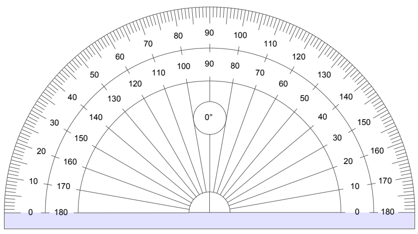
If A-B-C, then AB+BC=AC



3.3 – The Protractor and Angle Measure

Postulate 4: The Protractor Postulate – The rays in a half-rotation can be numbered from 0 to 180 so that positive number differences measure angles.

Definitions: An angle is



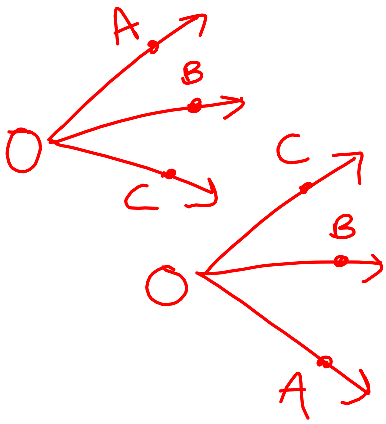
Acute iff it is less than 90°.

Right iff it is 90°.

Obtuse iff it is more than 90° but less than 180°.

Straight iff it is 180°.

Def: Betweenness of Rays – A ray is between two others in the same half-rotation iff its coordinate is between their coordinates.
(More briefly, OA-OB-OC iff a<b<c or a>b>c.)



Theorem 2: The Betweenness of Rays Theorem –
If OA-OB-OC, then ∠AOB+∠BOC=∠AOC.

3.4 – Bisection

Def: A point is on the midpoint of a line segment iff it divides the line segment into two equal segments.

Def: A line bisects an angle iff it divides the angle into two equal angles.

Def: Two objects are congruent if and only if they coincide exactly when superimposed.

Def: A corollary is a theorem that can be easily proved as a consequence of a postulate or another theorem.

Corollary to the Ruler Postulate: A line segment has exactly one midpoint.

Corollary to the Protractor Postulate: An angle has exactly one ray that bisects it.

3.5 – Complementary and Supplementary Angles

Def: Two angles are complementary iff their sum is 90°.

Def: Two angles are supplementary iff their sum is 180°.

Theorem 3: Complements of the same angle are equal. (proved on p.106)

Theorem 4: Supplements of the same angle are equal.

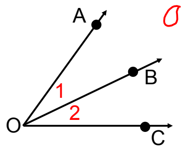
$\angle 1 + \angle AOD = 180° \rightarrow \angle AOD = 180° - \angle 1$

$\angle 2 + \angle AOD = 180° \rightarrow \angle AOD = 180° - \angle 2$

$180° - \angle 1 = 180° - \angle 2$

$-\angle 1 = -\angle 2$

$\angle 1 = \angle 2$



$\overrightarrow{OA} - \overrightarrow{OB} - \overrightarrow{OC}$

$\Rightarrow \angle AOB + \angle BOC = \angle AOC$

$\boxed{\angle 1 + \angle 2 = \boxed{\angle AOC}}$

In the figure, ∠1 and ∠2 are both complements of ∠AOC.

44. What else is true? $\overrightarrow{OB}$ bisects $\angle AOC$

$\boxed{\angle 1 = \angle 2;\ \angle 1 + \angle AOC = 90°,\ \angle 2 + \angle AOC = 90°}$

45. Is it possible to figure out the size of each angle in the figure without measuring them?

$$\begin{bmatrix} x + y = z \\ x = y \\ x + z = 90 \end{bmatrix}$$

$\angle 1 + \angle AOC - \angle 1 = 90° - \angle 1$

$\angle AOC = \boxed{90° - \angle 1}$

$\angle 1 + \angle 2 = 90° - \angle 1$

$+\angle 1 \qquad +\angle 1$

$2 \cdot \angle 1 + \angle 2 = 90°$

$2 \cdot \angle 1 + \angle 1 = 90°$

$3 \cdot \angle 1 = 90°$

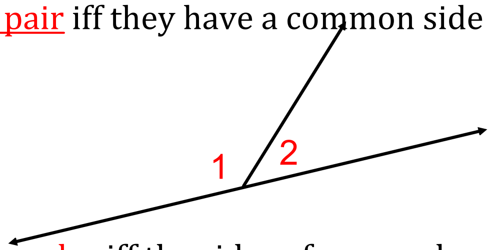
$\boxed{\angle 1 = 30°}$

$\boxed{\angle 2 = 30°}$

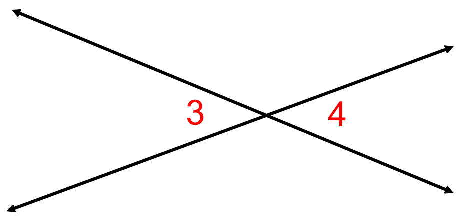
$\angle AOC = 60°$

3.6 – Linear Pairs and Vertical Angles

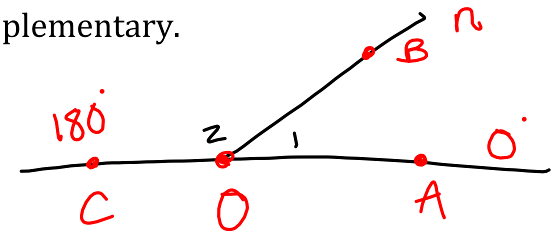
Def: Two angles are a linear pair iff they have a common side and their other sides are opposite rays.



Def: Two angles are vertical angles iff the sides of one angle are opposite rays to the sides of the other.



Theorem 5: The angles in a linear pair are supplementary.



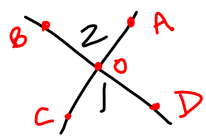
Given: ∠1 and ∠2 are a linear pair.

Prove: ∠1 and ∠2 are supplementary.

Proof:

Statements	**Reasons**
1. ∠1 and ∠2 are a linear pair.	Given
2. Rays OA and OC are opposite rays.	If two angles are a linear pair, they have a common side and their other sides are opposite rays. (def'n of linear pair)
3. Let the coordinates of OA, OB, and OC be 0, n, and 180.	Protractor postulate
4. ∠1=n-0=n° and ∠2=(180-n)°	Protractor Postulate
5. ∠1+∠2= n°+(180-n)°= 180°	substitution & Addition
6. ∠1 and ∠2 are supplementary.	Two angles are supplementary if their sum is 180°. (def'n of supp. ∠'s)

Theorem 6: Vertical angles are equal.



Given: ∠1 & ∠2 are vertical ∠'s.

To Prove: ∠1 = ∠2

Proof

Statements	Reasons
1. ∠1 & ∠2 are vertical ∠'s	Given
2. $\overrightarrow{OA}$ & $\overrightarrow{OC}$ are opposite rays $\overrightarrow{OB}$ & $\overrightarrow{OD}$ are opposite rays	definition of vertical ∠'s
3. ∠2 and ∠AOD form a linear pair ∠1 and ∠AOD form a linear pair	definition of linear pair
4. ∠2 & ∠AOD are supplementary ∠1 & ∠AOD are supplementary	Angles in a linear pair are supplementary
5. ∠1 = ∠2	Supplements of the same angle are equal

3.7 – Perpendicular and Parallel Lines

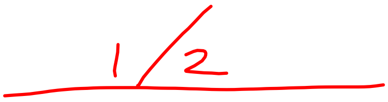
Def: Two lines are perpendicular iff they form a right angle.

Theorem 7: Perpendicular lines form four right angles.

Corollary to the definition of a right angle: All right angles are equal.

Theorem 8: If the angles in a linear pair are equal, then their sides are perpendicular.

Def: Two lines are parallel iff they lie in the same plane and do not intersect.



$\angle 1 + \angle 2 = 180°$ $\angle 1 = 90°$

$\angle 1 = \angle 2$

$\angle 1 + \angle 1 = 180°$