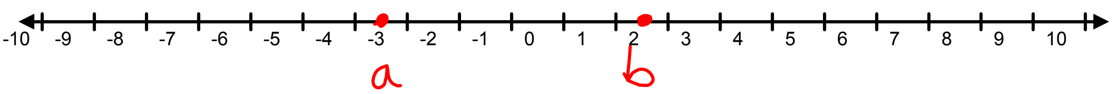
Chapter 4 – Congruence

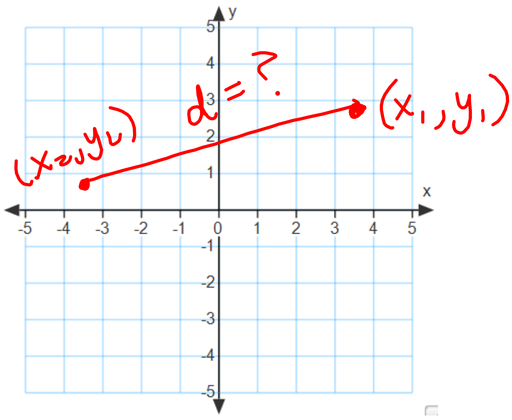
4.1 – Coordinates & Distance

distance between a & b is:
$|a-b| = |b-a|$

one-dimensional coordinate system:



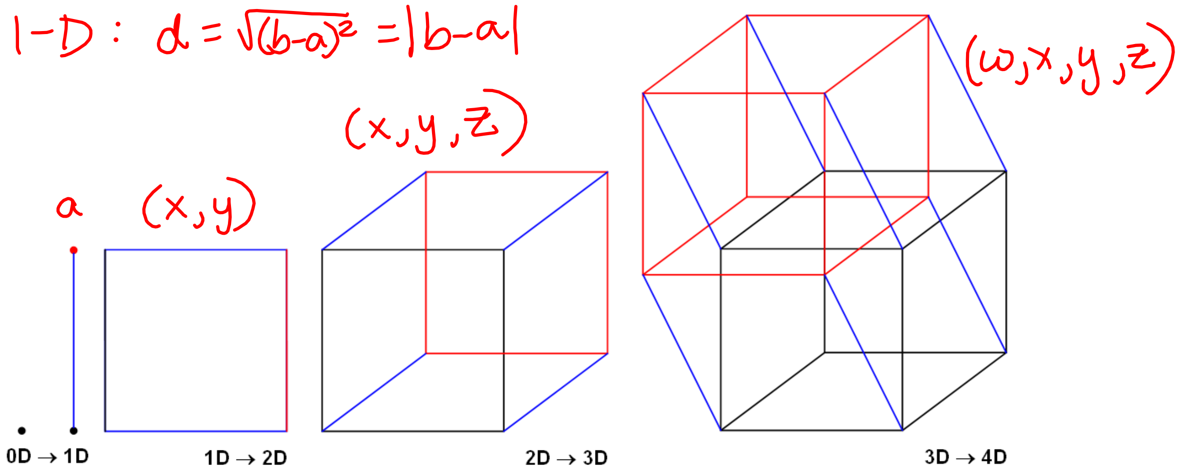
two-dimensional coordinate system:



Origin, axes, quadrants, coordinates

Distance formula:
The distance between the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

in 1-D: $d = \sqrt{(b-a)^2} = |b-a|$



distance in 3-D: $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

distance in 4-D: $d = \sqrt{(w_2-w_1)^2 + (x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

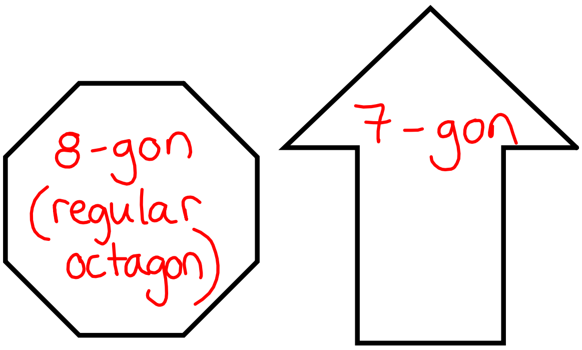
distance in 26-D: $d = \sqrt{(a_2-a_1)^2 + (b_2-b_1)^2 + \cdots + (z_2-z_1)^2}$

4.2 – Polygons and Congruence

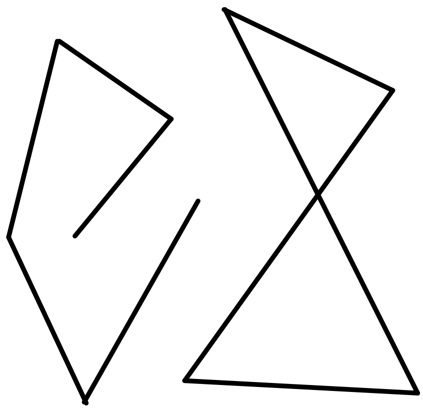
Def: A polygon is a connected set of at least three line segments in the same plane such that each segment intersects exactly two others, one at each endpoint.

The line segments are the sides of the polygon, and the endpoints are its vertices. The number of sides and vertices is always the same, and the polygon is referred to as an "*n*-gon" if it has *n* sides and *n* vertices.

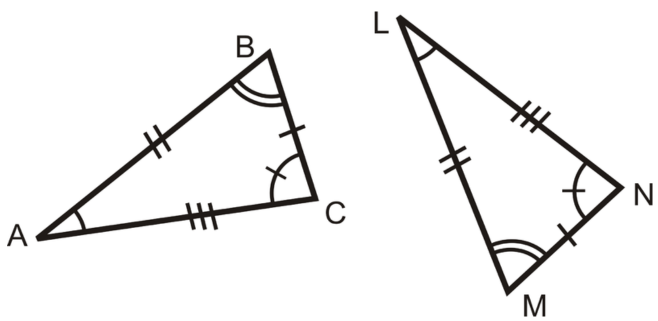
Polygons:



Not Polygons:



Def: Two triangles are congruent iff there is a correspondence between their vertices such that all of their corresponding sides and angles are equal.



$\triangle BCA \cong \triangle MNL$

$AC = LN$

$BC = MN$

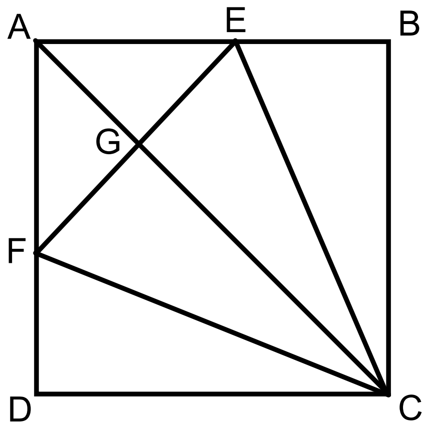
$\angle A = \angle L$

...

$\Delta ABC \cong \Delta LMN$

$\cong$ means "is congruent to"

$ABC \leftrightarrow LMN$



16. ΔFEC
ΔAFE

Name the triangles that appear to be congruent to the following triangles.

12. ΔAFG ≅ΔAEG

13. ΔACD ≅ΔACB ≅ΔCAB

14. ΔCDF ≅ΔCBE

15. ΔACE ≅ ΔACF

16. Name a triangle that is not congruent to any other triangle in the figure.

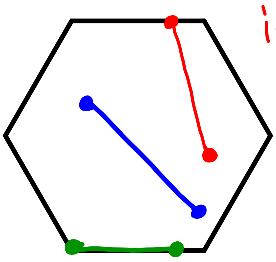
Corollary to the definition of congruent triangles: Two triangles congruent to a third triangle are congruent to each other.

55. Give the reasons for the statements in the proof.
Given: $\Delta ABC \cong \Delta XYZ$ and $\Delta DEF \cong \Delta XYZ$
Prove: $\Delta ABC \cong \Delta DEF$

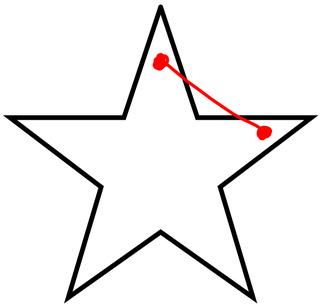
Proof:

Statements	**Reasons**
1. $\Delta ABC \cong \Delta XYZ$ and $\Delta DEF \cong \Delta XYZ$	Given
2. $\angle A = \angle X,\ \angle B = \angle Y, \angle C = \angle Z$ $AB = XY, BC = YZ, AC = XZ$ $\angle D = \angle X,\ \angle E = \angle Y, \angle F = \angle Z$ $DE = XY, EF = YZ, DF = XZ$	corresponding parts of congruent Δ's are equal
3. $\angle A = \angle D,\ \angle B = \angle E, \angle C = \angle F$ $AB = DE, BC = EF, AC = DF$	Substitution
4. $\Delta ABC \cong \Delta DEF$	corresponding parts of ≅ Δ's are =

Convex – every line segment connecting 2 points lies entirely inside the polygon

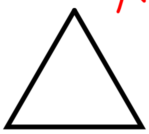


Concave – at least one line segment connecting 2 points goes outside the polygon

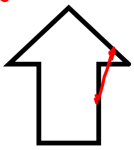


Convex or concave?

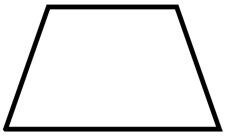
convex



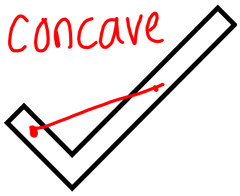
concave



convex



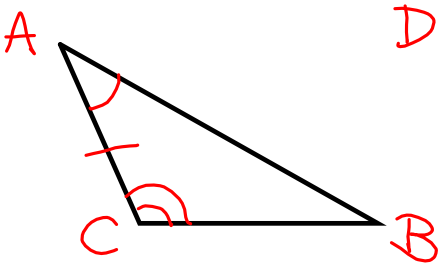
concave

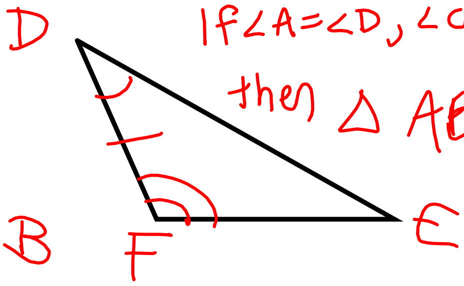


4.3 – ASA and SAS Congruence

Postulate 5: The ASA Postulate

If two angles and the included side of one triangle are equal to two andles and the included side of another triangle, the triangles are congruent.

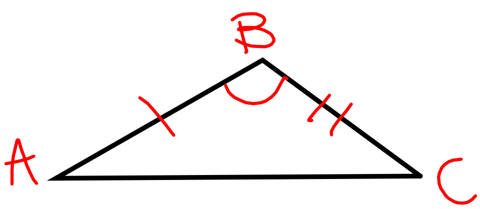


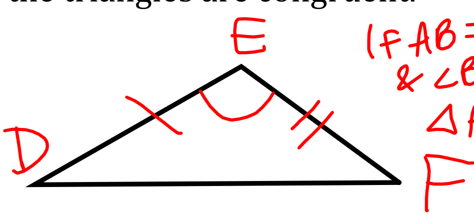


If $\angle A = \angle D$, $\angle C = \angle F$, & $AC = DF$ then $\triangle ABC \cong \triangle DEF$

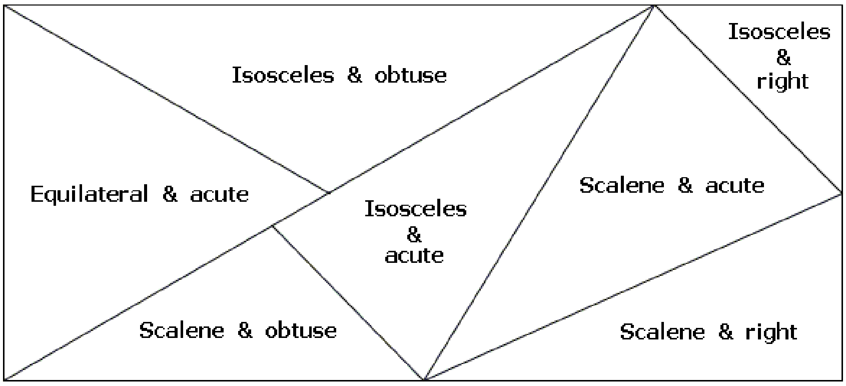
Postulate 6: The SAS Postulate

If two sides and the included angle of one triangle are equal to two sides and the included angle of another triangle, the triangles are congruent.





If $AB = DE$, $BC = EF$, & $\angle B = \angle E$, then $\triangle ABC \cong \triangle DEF$.



16. Equilateral – △ w/ all equal side lengths
17. Acute – △ w/ all angles less than 90° (all ∠'s acute)
18. Isosceles – △ w/ two equal sides
19. Obtuse – △ w/ one obtuse angle (greater than 90°)
20. Scalene – △ w/ no equal side lengths
21. Right – △ w/ one 90° (right) angle