

**Ch 4 - Congruence**

Def: A **polygon** is a connected set of at least three line segments in the same plane such that each segment intersects exactly two others, one at each endpoint.

The line segments are the sides of the polygon, and the endpoints are its vertices. The number of sides and vertices is always the same, and the polygon is referred to as an " $n$ -gon" if it has  $n$  sides and  $n$  vertices.

Def: Two triangles are **congruent** iff there is a correspondence between their vertices such that all of their corresponding sides and angles are equal.

**"Corresponding parts of congruent triangles are equal."**

Corollary to the definition of congruent triangles: Two triangles congruent to a third triangle are congruent to each other.

**Postulate 5: The ASA Postulate**

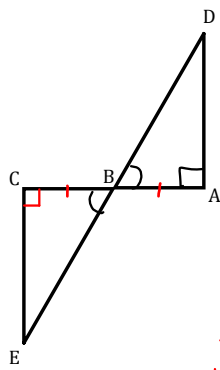
If two angles and the included side of one triangle are equal to two angles and the included side of another triangle, the triangles are congruent.

**Postulate 6: The SAS Postulate**

If two sides and the included angle of one triangle are equal to two sides and the included angle of another triangle, the triangles are congruent.

**4.4 - Congruence Proofs**

Def: Corresponding parts of congruent triangles are equal.



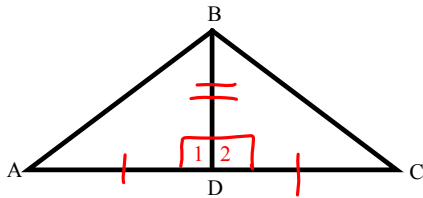
Given:  $CA \perp AD, CB = BA,$   
 $\angle C$  is a right angle and  
 $\angle CBE$  and  $\angle DBA$  are vertical angles.

Prove:  $CE = AD$

Proof

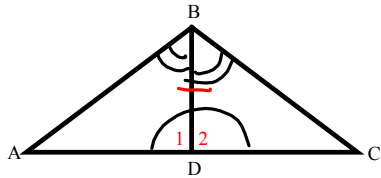
1.  $CA \perp AD$  Given
2.  $\angle A$  is a right angle Perpendicular lines form right angles
3.  $\angle C$  is a right angle given
4.  $\angle C = \angle A$  all right angles are equal
5.  $\angle CBE$  &  $\angle DBA$  are vertical angles Given
6.  $\angle CBE = \angle DBA$  vertical angles are equal
7.  $CB = BA$  Given
8.  $\triangle CBE \cong \triangle ABD$  ASA postulate
9.  $CE = AD$  corresponding parts of congruent  $\triangle$ 's are equal

**4.4 - Congruence Proofs, cont.**



Given: D is the midpoint of AC  
 $BD \perp AC$

2. Why is  $AD=DC$ ? a midpoint of a line segment divides it into 2 equal segments
3. Why are  $\angle 1$  and  $\angle 2$  right angles? perpendicular lines form right angles
4. Why is  $\angle 1 = \angle 2$ ? all right angles are equal
5. Why is  $BD=BD$ ? reflexive property
6. Why is  $\triangle ABD \cong \triangle CBD$ ? SAS
7. Why is  $\angle BAD = \angle BCD$ ? corresponding parts of congruent triangles are equal



Given:  $\angle 1 = \angle 2$   
 $BD = CD$

9. Why is  $BD = BD$ ?

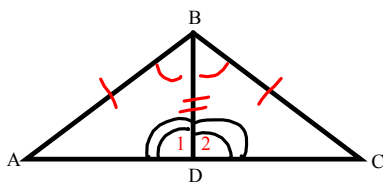
reflexive property

10. Why is  $\triangle ABD \cong \triangle CBD$ ?

ASA

11. Why is  $BA = BC$ ?

corresponding parts of congruent triangles are equal



Given:  $BA = BC$   
 $BD$  bisects  $\angle ABC$

13. Why is  $\angle ABD = \angle CBD$ ?

angle bisector divides an angle into 2 equal angles

14. Why is  $BD = BD$ ?

reflexivity

15. Why is  $\triangle ABD \cong \triangle CBD$ ?

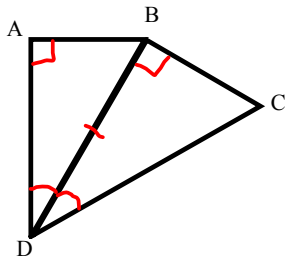
SAS

16. Why is  $\angle 1 = \angle 2$ ?

corresponding parts of congruent triangles are equal

17. If  $\angle 1$  and  $\angle 2$  are a linear pair, why is  $BD \perp AC$ ?

If angles in a linear pair are equal, their sides are perpendicular



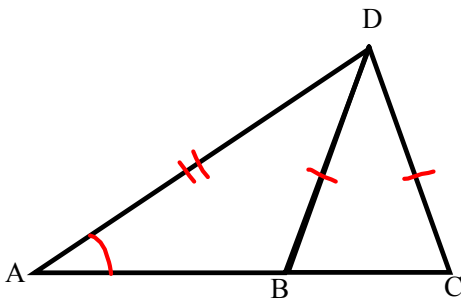
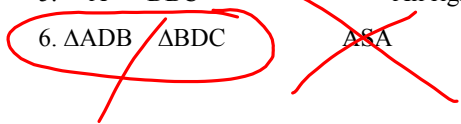
**What is wrong with this proof?**

*Given:* DB bisects  $\angle ADC$   
 $\angle A$  and  $\angle B$  are right angles.

*Prove:*  $\triangle ADB \cong \triangle BDC$

*Proof:*

<u>Statements</u>	<u>Reasons</u>
1. DB bisects $\angle ADC$	Given.
2. $\angle ADB = \angle BDC$	If an angle is bisected, it is divided into two equal angles.
3. $BD = BD$	Reflexive.
4. $\angle A$ and $\angle B$ are right angles	Given
5. $\angle A = \angle B$	All right angles are equal.
6. $\triangle ADB \cong \triangle BDC$	<del>ASA</del>



**What is wrong with this proof?**

*Given:*  $DB = DC$

*Prove:*  $AB = AC$

*Proof:*

Statements

1.  $DB = DC$
2.  $AD = AD$
3.  $\angle DAB = \angle DAC$
4.  $\triangle DAB \cong \triangle DAC$
5.  $AB = AC$

Reasons

- Given  
 Reflexive  
 Reflexive  
~~SAS~~  
 Corresponding parts of congruent triangles are congruent

NOT a congruence  
 ↓  
 ASS

