5.1 – Properties of Inequality

The "Three Possibilities" Property: either $a>b$, $a=b$, or $a<b$

The Transitive Property: If $a>b$ and $b>c$, then $a>c$ $a > b > c$

The Addition Property: If $a>b$, then $a+c>b+c$

The Subtraction Property: If $a>b$, then $a-c>b-c$

The Multiplication Property: If $a>b$ and $c>0$, then $ac>bc$

The Division Property: If $a>b$ and $c>0$, then $a/c>b/c$

The Addition Theorem of Inequality: If $a>b$ and $c>d$, then $a+c>b+d$

The "Whole Greater than Part" Theorem: If $a>0$, $b>0$, and $a+b=c$, then $c>a$ and $c>b$

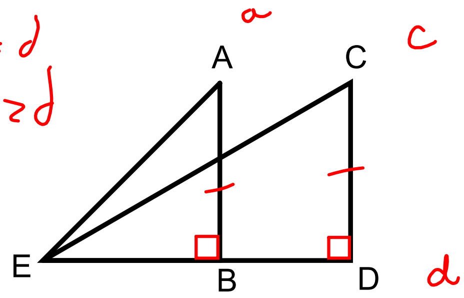
Proof:

Statements	Reasons
1. $a>0$ and $b>0$ $+b$ $+b$ $+a$ $+a$	Given
2. $a+b>b$ and $a+b>a$	Addition
3. $a+b=c$	Given
4. $c>b$ and $c>a$	Substitution (#3 into #2)

47.

Given: AB=CD; EA-EC-ED
Prove: ∠AED>∠CED

iff $a < c < d$
or $a > c > d$

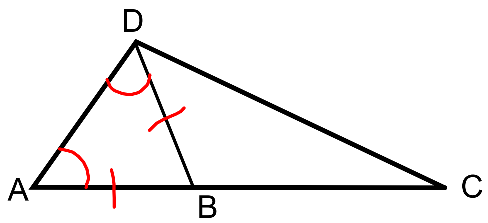


Proof:

Statements	**Reasons**
1. EA-EC-ED	Given
2. ∠AED=∠AEC+∠CED	Betweenness of Rays Theorem
3. ∠AEC>0	Betweenness of Rays definition
4. ∠AED>∠CED	Whole greater than part

48.

Given: A-B-C; ∠ADB=∠DAB
Prove: AC>DB



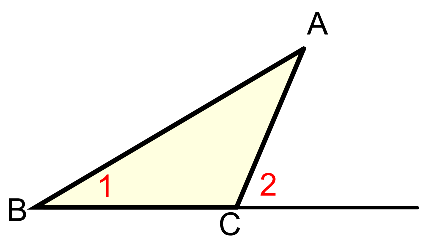
Proof:

Statements:	**Reasons:**
1. A-B-C, ∠ADB=∠DAB	Given
2. AB+BC=AC	Betweenness of Points Theorem
3. AB=DB	If 2 angles in a △ are =, then the sides opposite them are =
4. DB+BC=AC	Substitution (#3 into #2)
5. AC>DB	Whole greater than part

5.2 – The Exterior Angle Theorem

Def: An exterior angle of a triangle is an angle that forms a linear pair with an angle of the triangle.

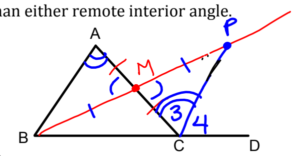
In ΔABC, exterior ∠2 forms a linear pair with ∠ACB.
The other two angles of the triangle, ∠1 (∠B) and ∠A are called remote interior angles with respect to ∠2.



Theorem 12: The Exterior Angle Theorem

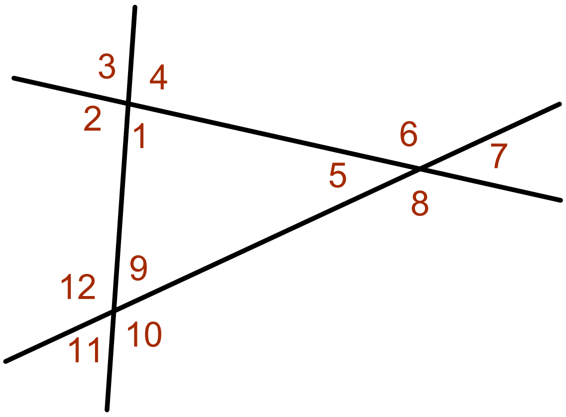
An Exterior angle of a triangle is greater than either remote interior angle.

Given: ∠ACD is an exterior angle of ΔABC.
Prove: ∠ACD>∠A and ∠ACD>∠B



Proof:

Statements	**Reasons**
1. ∠ACD is an exterior angle of ΔABC	Given
2. Let M be the midpoint of AC	Every line segment has a unique midpoint
3. AM=MC	Midpoint divides a segment into 2 equal segments
4. Draw line BM	2 points define a line
5. Choose P on line BM so that MB=MP	Ruler Postulate
6. Draw CP	2 points define a line
7. ∠AMB=∠CMP	Vertical ∠'s are =
8. ΔAMB≅ΔCMP	SAS
9. ∠A=∠3	corresponding parts of congruent Δ's are equal
10. ∠ACD=∠3+∠4	Betweenness of Rays Theorem
11. ∠ACD>∠3	Whole greater than part
12. ∠ACD>∠A	Substitution
13. ∠ACD>∠B	An exterior ∠ is larger than either remote interior angle



Find each of the following sums.

26. $\angle 1+\angle 2+\angle 3+\angle 4$

$360°$

27. $\angle 1+\angle 2+\angle 3+\angle 4+\angle 5+\angle 6+\angle 7+\angle 8+\angle 9+\angle 10+\angle 11+\angle 12$

$3(360°) = 1080°$

28. $\angle 1+\angle 5+\angle 9$

$180°$

29. $\angle 3+\angle 7+\angle 11$

$180°$

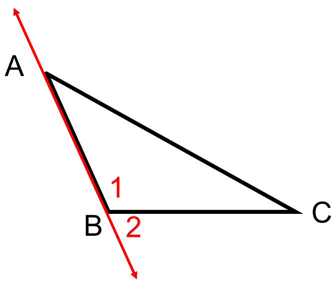
30. $\angle 2+\angle 4+\angle 6+\angle 8+\angle 10+\angle 12$

$1080° - 180° - 180° = 720°$

31. What does the result in exercise 30 indicate about the sum of the exterior angles of a triangle?

sum to 720°

After proving the Exterior Angle Theorem, Euclid proved that, in any triangle, the sum of any two angles is less than 180°. Prove that, in ΔABC, $\angle A+\angle B<180°$ by giving a reason for each of the following statements.



39. Draw line AB.

2 points define a line

40. $\angle 2$ is an exterior angle of ΔABC.

it forms a linear pair w/∠1

41. $\angle 1$ and $\angle 2$ are supplementary.

∠'s in a linear pair are supplementary

42. $\angle 1+\angle 2=180°$.

supplementary ∠'s sum to 180°

43. $\angle 2>\angle A$

Exterior angle is greater than either remote interior angle

44. $\angle 1+\angle 2>\angle 1+\angle A$

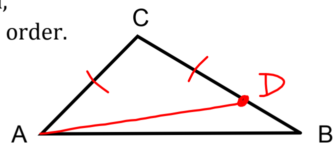
Addition

45. $180°>\angle 1+\angle A$, so $\angle 1+\angle A<180°$ $a<b \Leftrightarrow b>a$

↑ substitution (#42 into #44)

5.3 Triangle Side and Angle Inequalities

Theorem 13: If two sides of a triangle are unequal, the angles opposite them are unequal in the same order.



Given: ΔABC with BC>AC
Prove: ∠A>∠B

Proof:

Statements	**Reasons**
1. ΔABC with BC>AC	Given
2. Choose D on CB so that CD=CA	Ruler Postulate
3. Draw AD	2 points define a line
4. ∠1=∠2	
5. ∠CAB=∠1+∠DAB	
6. ∠CAB>∠1	
7. ∠CAB>∠2	
8. ∠2>∠B	
9. ∠CAB>∠B	