5.1 – Properties of Inequality

The "Three Possibilities" Property: either a>b, a=b, or a<b

The Transitive Property: If a>b and b>c, then a>c   a>b>c

The Addition Property: If a>b, then a+c>b+c

The Subtraction Property: If a>b, then a-c>b-c

The Multiplication Property: If a>b and c>0, then ac>bc

The Division Property: If a>b and c>0, then a/c>b/c

The Addition Theorem of Inequality: If a>b and c>d, then a+c>b+d

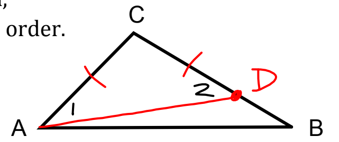
The "Whole Greater than Part" Theorem: If a>0, b>0, and a+b=c, then c>a and c>b

Def: An exterior angle of a triangle is an angle that forms a linear pair with an angle of the triangle.

Theorem 12: The Exterior Angle Theorem
An Exterior angle of a triangle is greater than either remote interior angle.

5.3 Triangle Side and Angle Inequalities

Theorem 13: If two sides of a triangle are unequal, the angles opposite them are unequal in the same order.

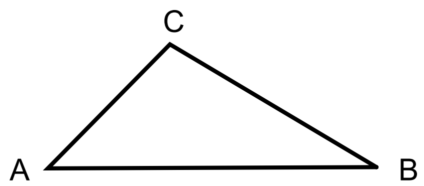


Given: ΔABC with BC>AC
Prove: ∠A>∠B

Proof:

Statements	Reasons
1. ΔABC with BC>AC	Given
2. Choose D on CB so that CD=CA	Ruler Postulate
3. Draw AD	2 points define a line
4. ∠1=∠2	If a Δ has 2 equal sides, then the ∠'s opposite them are =
5. ∠CAB=∠1+∠DAB	Betweenness of Rays Theorem
6. ∠CAB>∠1	Whole greater than Part
7. ∠CAB>∠2	Substitution (#4 into #6)
8. ∠2>∠B	An exterior angle is greater than either remote interior angle
9. ∠CAB>∠B	Transitive Property of Inequality

Theorem 14: If two angles of a triangle are unequal, the sides opposite them are unequal in the same order.



Given: ΔABC with ∠A>∠B
Prove: BC>AC

Proof:

Statements **Reasons**

Suppose that BC is not longer than AC

4. Then either BC=AC or BC<AC — 5. Three Possibilities Property

6. If BC=AC, then ∠A=∠B — 7. If 2 sides of a Δ are equal, then the ∠'s opposite them are =

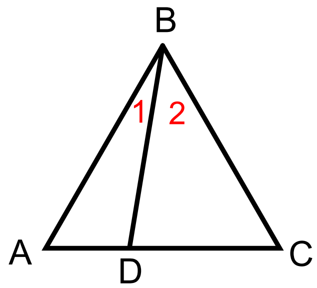
8. This contradicts the hypothesis (given) that ∠A>∠B.

9. If BC<AC, then ∠A<∠B — 10. Thm 13 If 2 sides of a Δ are unequal, then the ∠'s opposite them are unequal in the same order

11. This also contradicts the hypothesis that ∠A>∠B

12. Therefore, what we suppose is false and BC>AC.

Given: ΔABC is equilateral.
Prove: BD>DC

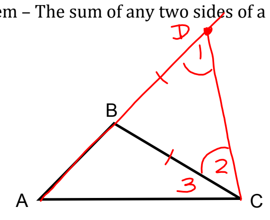


Proof:

Statements	**Reasons**
45. ∠C=∠ABC	Equalaterateral Δ's are equiangular
46. ∠ABC=∠1+∠2	Betweenness of Rays Theorem
47. ∠ABC>∠2	Whole greater than part
48. ∠C>∠2.	Substitution (#45 into 47)
49. BD>DC	If 2 angles in a triangle are unequal, then the angles opposite them are unequal in the same order

5.4 The Triangle Inequality Theorem

Theorem 15: The Triangle Inequality Theorem – The sum of any two sides of a triangle is greater than the third side.



Given: ABC is a triangle

Prove: AB+BC>AC

Proof:

Statements	**Reasons**
1. ABC is a triangle	Given
2. Draw line AB	2 pts define a line
3. Choose D beyond B on line AB so that BD=BC	Ruler Postulate
4. Draw CD	2 pts define a line
5. ∠1=∠2	If 2 sides of a Δ are =, then the ∠'s opposite them are =
6. ∠ACD=∠2+∠3	Betweenness of Rays Theorem
7. ∠ACD>∠2	Whole greater than Part
8. ∠ACD>∠1	Substitution (#5 into #7)
9. In ΔACD, AD>AC	If 2 angles of a triangle are unequal, then the sides opposite them are unequal in the same order
10. AB+BD=AD	Betweenness of Points Theorem
11. AB+BD>AC	Substitution (# 10 into # 9)
12. AB+BC>AC	Substitution (# 3 into #11)

SAT Problem: ~~3~~, ~~4~~, ~~5~~, 6

If x is an integer and $2<x<7$, how many different triangles are there with sides of lengths 2, 7, and x?

15. Could $x=3$? Why or why not?

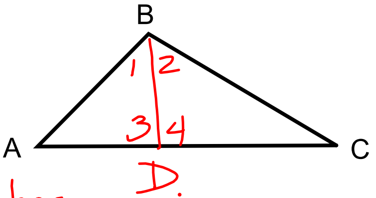
no, $2+3=5 \not> 7$

16. What do you think is the answer to the problem? Explain.

1 triangle $(x=6)$

Heron's Proof of the Triangle Inequality

Given: ABC is a triangle.
Prove: AB+BC>AC



Proof:

Statements	**Reasons**
24. Let BD bisect ∠ABC	every angle has a unique angle bisector
25. ∠1=∠2	an angle bisector divides an angle into 2 equal angles
26. ∠3>∠2 and ∠4>∠1	Exterior angle is greater than either remote interior angle
27. ∠3>∠1 and ∠4>∠2	substitution (#25 into 26)
28. AB>AD and BC>DC	If 2 ∠'s in a △ are unequal the sides opposite them are unequal in the same order
29. AB+BC>AD+DC	Addition Theorem of Inequality
30. AD+DC=AC	Betweenness of Points Theorem
31. AB+BC>AC	substitution (#30 into 29)