7.1 – Quadrilaterals

Def: A diagonal of a polygon is a line segment that connects any two nonconsecutive vertices.

Theorem 24: The sum of the angles of a quadrilateral is 360°.

Def: A rectangle is a quadrilateral each of whose angles is a right angle.

Corollary to Theorem 24: A quadrilateral is equiangular iff it is a rectangle.

Given: ABCD with A= B= C= D.
Prove: ABCD is a rectangle.

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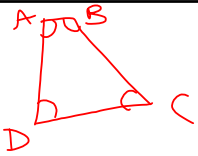
Prove: A= B= C= D.



ABCD ▱

⇒ 1. $\angle A = \angle B = \angle C = \angle D$ — Given

2. $\angle A + \angle B + \angle C + \angle D = 360°$ — Quadrilateral Sum Theorem

3. $4\angle A = 360°$ — Substitution & Simplification

4. $\angle A = 90°$ — Division

5. $\angle B = \angle C = \angle D = 90°$ — Substitution

6. $\angle A, \angle B, \angle C, \& \angle D$ are right $\angle$'s — 90° angles are right angles

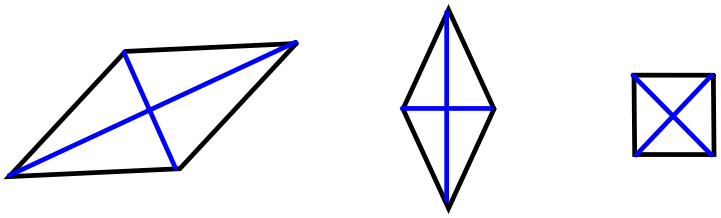
7. ABCD is a rectangle. — Def'n of rectangle

⇐ 1. ABCD is a rectangle — Given

2. $\angle A, \angle B, \angle C, \& \angle D$ are right angles — all angles in a rectangle are right $\angle$'s

3. $\angle A = \angle B = \angle C = \angle D$ — all right $\angle$'s are =

Each of the figures below is a rhombus.



What seems to be true about

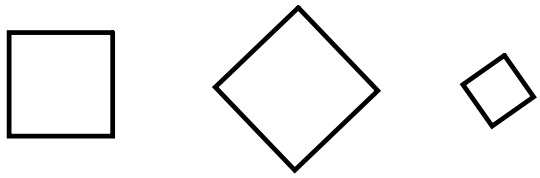
19. the sides of a rhombus?

all equal

20. the diagonals of a rhombus?

perpendicular & bisect each other

Each of the figures below is a square.



What property do you think squares have in common with

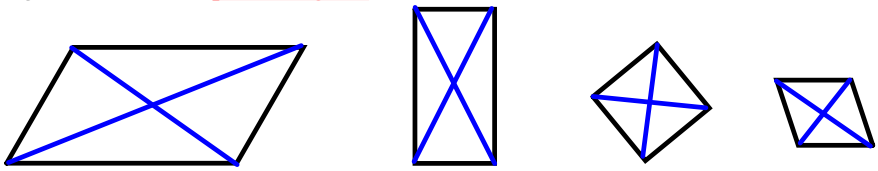
21. rectangles

all right ∠'s

22. rhombuses?

all sides are equal

Each of the figures below is a parallelogram.



What seems to be true about

23. The opposite sides of a parallelogram?

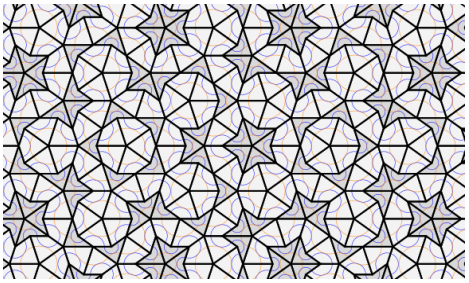
equal & parallel

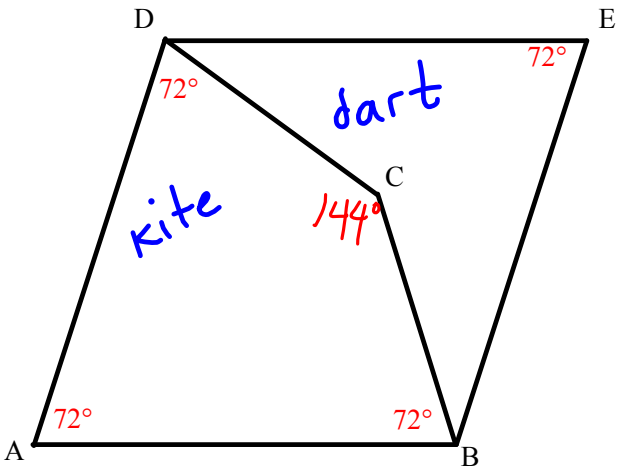
24. The opposite angles of a parallelogram?

equal

25. The diagonals of a parallelogram?

bisect each other





28. Which Penrose tile is convex?

Kite

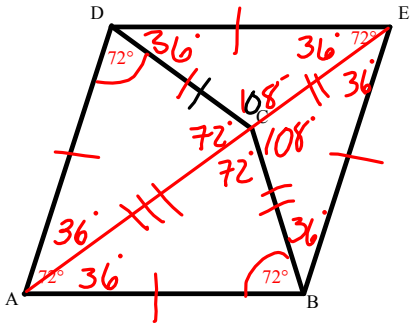
29. Find the measure of BCD.

$\angle BCD + 72^\circ + 72^\circ + 72^\circ = 360^\circ$

$\angle BCD = 144^\circ$

30. Draw AC and CE.

The figure is drawn so that AB=BE=ED=DA and CB=CE=CD.



31. How do you know that ΔADC ΔABC and ΔEDC ΔEBC?

Find the measures of the rest of the angles.

32. What do all four triangles in the figure have in common?

Isosceles

33. What seems to be true about points A, C, and E?

collinear

34. Why?

$\angle ACD$ & $\angle DCE$ form a linear pair (sum to 180° w/ shared side)

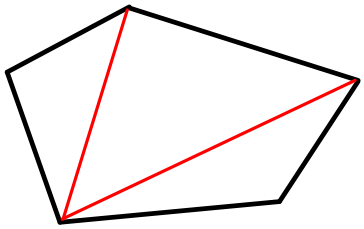
35. Does the figure appear to have line symmetry? Why or why not?

yes with respect to AE (AB=BE=ED=AD & DC=CB)

36. If a quadrilateral is equilateral, does it follow that it is also equiangular? Why or why not?

NO – rhombuses are equilateral, but not necessarily equiangular

In the figure below, a pentagon has been divided into triangles by the diagonals from one vertex.

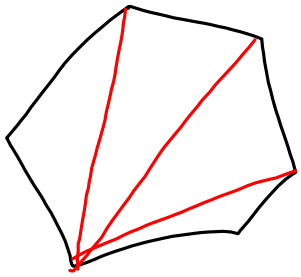


38. How many sides does a pentagon have, 5

how many diagonals were drawn, 2

and how many triangles were formed? 3

39. Draw a hexagon and the diagonals from one vertex.

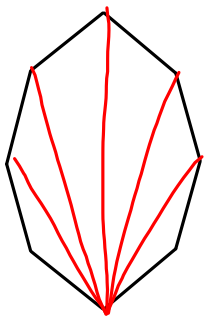


40. How many sides does a hexagon have, 6

how many diagonals did you draw, 3

and how many triangles were formed? 4

41. Draw the diagonals from one vertex for the given figure.



42. How many sides does the polygon have, 8 (octagon)
how many diagonals did you draw, 5
and how many triangles were formed? 6

In general, if a polygon has n sides, in terms of n,

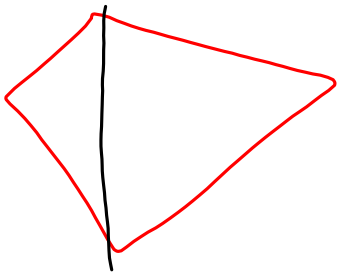
43. how many diagonals can be drawn from one vertex?

$n-3$

44. how many triangles do these diagonals form?

$n-3+1 = n-2$

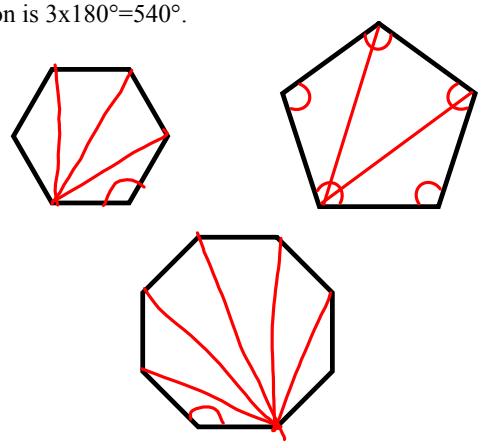
45. Show that your answers are correct for a quadrilateral.



$n=4$
1 diagonal $= 4-3$
2 triangles $= 4-2$ ✓

The figure below suggests that the sum of the angles of a pentagon is 3x180°=540°.

If the pentagon is equiangular, then each angle is 540°/5=108°.



46. What is the sum of the angles of a hexagon

$180^\circ \times 4 = 720^\circ$

47. If the hexagon is equiangular, how large is each angle?

$720^\circ \div 6 = 120^\circ$

48. What is the sum of the angles of an octagon?

$180^\circ \times 6 = 1080^\circ$

49. If the octagon is equiangular, how large is each angle?

$1080^\circ \div 8 = 135^\circ$

50. What, in terms of n, is the sum of the angles of an n-gon?

$180^\circ \times (n-2)$

51. If the n-gon is equiangular, how large is each angle in terms of n?

$$\frac{180^\circ (n-2)}{n}$$ = interior ∠ measure of a regular n-gon