Def: A parallelogram is a quadrilateral whose opposite sides are parallel.
Theorem 25: The opposite sides and angles of a parallelogram are equal.
Theorem 26: The diagonals of a parallelogram bisect each other.
Theorem 27: A quadrilateral is a parallelogram, if its opposite sides are equal.
Theorem 28: A quadrilateral is a parallelogram if its opposite angles are equal.
Theorem 29: A quadrilateral is a parallelogram if two opposite sides are both parallel and equal.
Theorem 30: A quadrilateral is a parallelogram if its diagonals bisect each other.

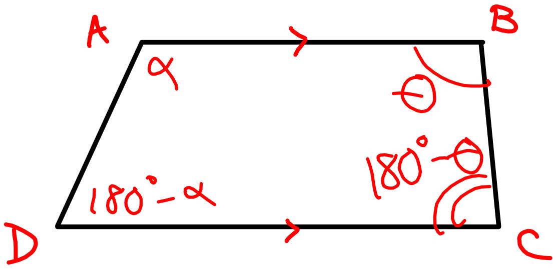
7.4 – Rectangles, Rhombuses, and Squares

Def: A square is a quadrilateral all of whose sides and angles are equal.
Every square is a rhombus.
Def: A rhombus is a quadrilateral all of whose sides are equal.
Theorem 31: All rectangles are parallelograms.
Theorem 32: All rhombuses are parallelograms.
Theorem 33: The diagonals of a rectangle are equal.
Theorem 34: The diagonals of a rhombus are perpendicular.

7.5 – Trapezoids

Def: A trapezoid is a quadrilateral that has exactly one pair of parallel sides.

The parallel sides are called the bases of the trapezoid, and the non-parallel sides are called its legs. The pairs of angles that include each base are called base angles.



In this trapezoid:

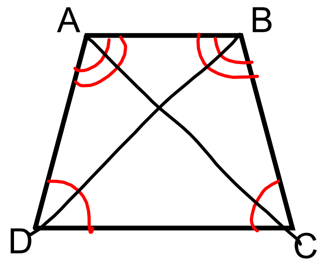
Sides AB and DC are bases.

Sides AD and BC are legs.

Angles A and B are one pair of base angles.

Angles D and C are another pair of base angles.

Def: An isosceles trapezoid is a trapezoid whose legs are equal.



Theorem 35: The base angles of an isosceles trapezoid are equal.
Given: ABCD is an isosceles trapezoid with bases AB and DC.
Prove: A= B and D= C

Theorem 36 : The diagonals of an isosceles trapezoid are equal.
Given: ABCD is an isosceles trapezoid with bases AB and DC.
Prove: DB=CA.

If a quadrilateral is a trapezoid, then its diagonals cannot bisect each other.

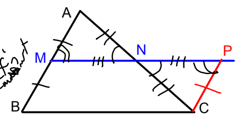
Given: ABCD is a trapezoid
Prove: AC and DB do not bisect each other.

7.6 – The Midsegment Theorem

Def: A midsegment of a triangle is a line segment that connects the midpoints of two of its sides.

Theorem 37 : The Midsegment Theorem – A midsegment of a triangle is parallel to the third side and half as long.

Given: MN is a midsegment of ΔABC.
Prove: MN||BC and MN=(1/2)BC.



1. M is midpoint of AB
   N is midpoint of AC — def'n of midpoint
2. AM = MB & AN = NC — midpoint divides a segment into 2 equal segments
3. extend MN to point P so that MN = NP — (Ruler postulate)
4. Draw CP — 2 pts define a line
5. ∠ANM = ∠CNP — vertical ∠'s are =
6. ΔANM ≅ ΔCNP — SAS
7. ∠AMN = ∠CPN; AM = CP — corresponding parts of ≅ Δ's are =
8. AB ∥ CP — equal alternate interior ∠'s mean lines are parallel
9. CP = MB — Substitution
10. BMPC is a parallelogram — a quadrilateral w/ an equal & parallel pair of opposite sides is a parallelogram
11. MN ∥ BC — Opposite sides of a parallelogram are parallel
12. BC = MP — parallelogram has equal opposite sides
13. MN + NP = MP — Betweenness of Points Theorem
14. MN + NB = BC — substitution
15. MN + MN = BC
    2 MN = BC — Substitution & Simplification
16. MN = ½ BC — Division

8.1 – Transformations

Def: A transformation is a one-to-one correspondence between two sets of points.

A translation slides an object a certain distance without turning it.

A reflection flips an object over a mirror line.

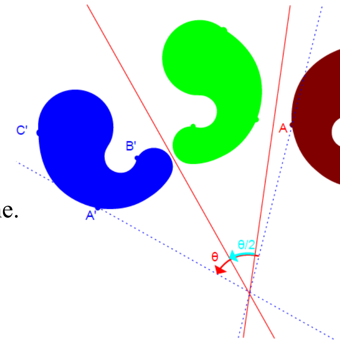
A rotation turns an object a certain number of degrees about a fixed point.

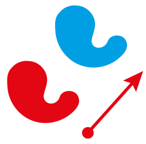
A dilation enlarges or reduces the size of an object.

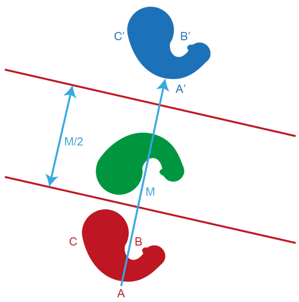
Def: an isometry is a transformation that preserves distance and angle measure.

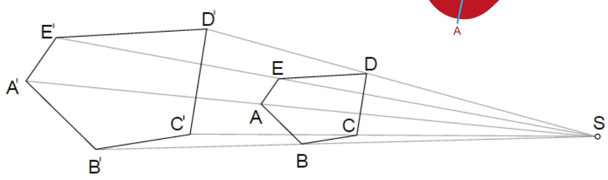
Translations, reflections, and rotations

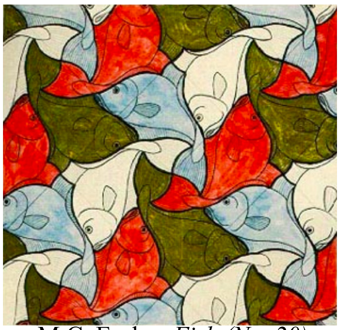
are all examples of isometries, but dilations are not.










M.C. Escher. *Fish (No. 20)*

What type of translation seems to relate

1. Two fish of the same color?

2. A pair of red and white fish?

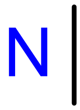
3. A pair of blue and white fish?

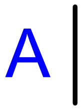
Are there any pairs of fish in the figure for which one fish of the pair seems to be

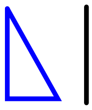
4. A dilation of the other?

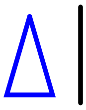
5. A reflection of the other?

Complete the figures by including the reflection image of the object through the mirror line.

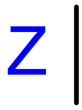
8. 

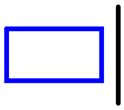
9. 

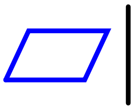
10. 

11. 

12. 

13. 

14. 

15. 

16. Which figures look the same as their mirror images?

17. What is it about these figures that causes them and their mirror images to look the same?

<u>8.2 – Reflections</u>

Def: The <u>reflection</u> of point P through line l is P itself if P lines on l. Otherwise, it is the point P' such that l is the perpendicular bisector of PP'.

<u>Construction 8</u>: To reflect a point through a line.

Def: A <u>translation</u> is the composite of two successive reflections through parallel lines.

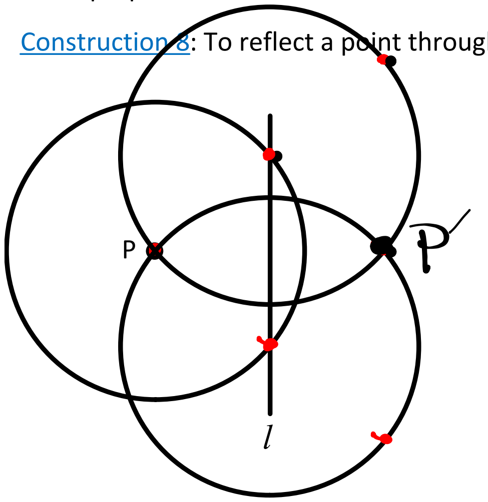
The distance between a point of the original figure and its translation image is called the *magnitude* of the translation.

Def: A <u>rotation</u> is the composite of two successive reflections through intersecting lines. The point in which the lines intersect is the *center* of rotation, and the measure of the angle through which a point of the original figure turns to coincide with its rotation image is called the *magnitude* of the rotation.

8.2 – Reflections

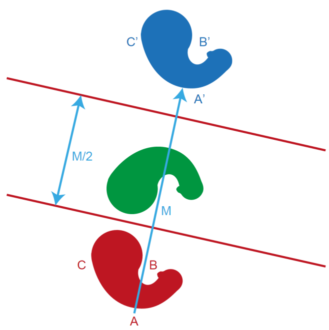
Def: The reflection of point P through line l is P itself if P lines on l. Otherwise, it is the point P' such that l is the perpendicular bisector of PP'.

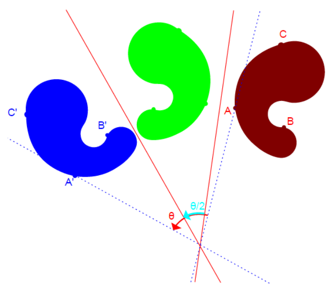
Construction 8: To reflect a point through a line.



Def: A translation is the composite of two successive reflections through parallel lines.

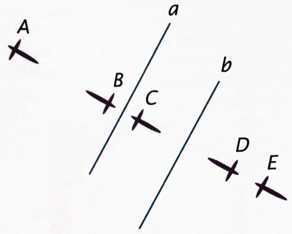
The distance between a point of the original figure and its translation image is called the *magnitude* of the translation.





Def: A rotation is the composite of two successive reflections through intersecting lines.
The point in which the lines intersect is the *center* of rotation, and the measure of the angle through which a point of the original figure turns to coincide with its rotation image is called the *magnitude* of the rotation.

In the figure below, $a \parallel b$ and birds A, B, D, and E are reflection images of bird C through either or both of the lines.



Which bird is the reflection image of

31. bird C through a?

32. bird B through b?

33. bird C through b?

34. bird D through a?

Which bird is the image of bird C as a result of successive reflections through

35. a and b?

36. b and a?

37. What transformation do exercises 35 and 36 illustrate?

31. B

32. E

33. D

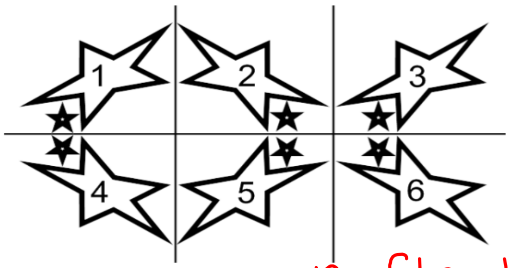
34. A

35. E

36. A

translations

Part IV – State the type of transformation that takes the following objects to their images. Choose your answer from the following: ***translation, reflection, rotation, dilation, glide reflection.***



51. What type of transformation takes object 1 to object 2? reflection

52. What type of transformation takes object 1 to object 3? translation

53. What type of transformation takes object 1 to object 4? reflection

54. What type of transformation takes object 1 to object 5? rotation

55. What type of transformation takes object 1 to object 6? glide reflection