

Ch 10 Review, pp. 421-424 #1-62

Def: The ratio of the number a to the number b is the number a/b .

A proportion is an equality between ratios. $a/b=c/d$

a, b, c, and d are called the *first, second, third, and fourth terms*.

The second and third terms, b and c, are called the means.

The first and fourth terms, a and d, are called the extremes.

$$\frac{a}{x} = \frac{x}{d}$$

The product of the means is equal to the product of the extremes.

If $a/b=c/d$, then $ad=bc$.

Def: The number b is the geometric mean between the numbers a and c if a, b, and c are positive and $a/b=b/c$.

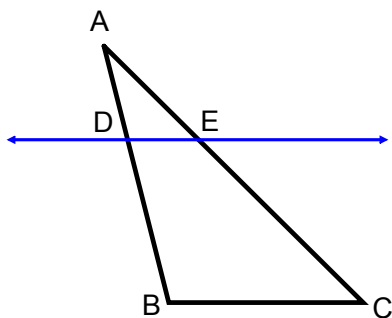
$$x^2 = ad \Rightarrow x = \sqrt{ad}$$

Def: Two triangles are similar iff there is a correspondence between their vertices such that their corresponding sides are proportional and their corresponding angles are equal.

10.3 - The Side-Splitter Theorem

Theorem 44 - The Side-Splitter Theorem

If a line parallel to one side of a triangle intersects the other two sides in different points, it divides the sides in the same ratio, that is, if in triangle ABC, $DE \parallel BC$, then $AD/DB=AE/EC$.



Corollary to the Side-Splitter Theorem:

If a line parallel to one side of a triangle intersects the other two sides in different points, it cuts off segments proportional to the sides, that is, $AD/AB=AE/AC$ and $DB/AB=EC/AC$

10.4 - AA Similarity

Theorem 45 - The AA Theorem - If two angles of one triangle are equal to two angles of another triangle, the triangles are similar.

Corollary to the AA Theorem - Two triangles similar to a third triangle are similar to each other.

Piero della Francesca, an important painter of the 15th century, was also a mathematician. In his book *On Perspective for Painting*, he proved the following theorem:

"If above a line divided into several parts a line be drawn parallel to it and from the points dividing the first line there be drawn lines which are concurrent, they will divide the parallel line in the same proportion as the given line."



View of an Ideal City, 1460

19. What does this theorem say about lines BC and HI?

they are parallel

20. What does the word "concurrent" mean?

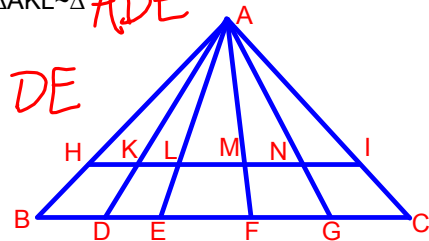
they share a point (or intersect)

21. Complete the similarity correspondences: $\triangle AHK \sim \triangle ABD$ and $\triangle AKL \sim \triangle ADE$

22. Complete the proportions: $HK/BD = AK/AD$ and $AK/AD = KL/DE$

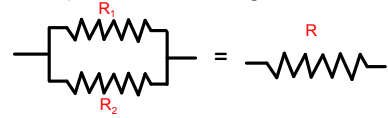
23. What proportion follows directly from these two proportions?

$$HK/BD = KL/DE$$



Electricians know that if two resistances R_1 and R_2 are "in parallel," they are equivalent to a single resistance R , where $R = (R_1 R_2) / (R_1 + R_2)$.

Prove that the figure below illustrates this equation by giving a reason for each of the following statements.



25. $\triangle EFC \sim \triangle ABC$ and $\triangle EFB \sim \triangle DCB$

AA Similarity

26. $R/R_1 = y/(x+y)$ and $R/R_2 = x/(x+y)$

$$\frac{R}{R_1} = \frac{y}{x+y} \quad \frac{R}{R_2} = \frac{x}{x+y}$$

27. $R/R_1 + R/R_2 = y/(x+y) + x/(x+y) = (y+x)/(x+y) = 1$

$$\frac{R}{R_1} + \frac{R}{R_2} = \frac{y}{x+y} + \frac{x}{x+y} = \frac{x+y}{x+y} = 1$$

28. $RR_2 + RR_1 = R_1 R_2$

$$RR_2 + RR_1 = R_1 R_2$$

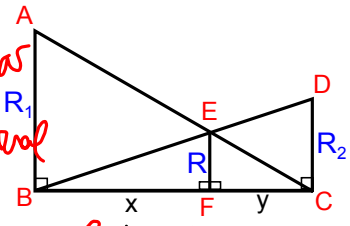
29. $R(R_2 + R_1) = R_1 R_2$

Distributive prop.

30. $R = (R_1 R_2) / (R_1 + R_2)$

$$R = \frac{R_1 R_2}{R_1 + R_2} \quad \text{Division}$$

Corresponding parts of similar Δ 's are proportional

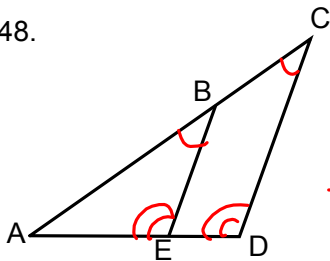


$$R_1 R_2 \cdot \left(\frac{R}{R_1} + \frac{R}{R_2} \right) = (1) \cdot R_1 R_2$$

$$\frac{R_1 R_2 R}{R_1} + \frac{R_1 R_2 R}{R_2} = R_1 R_2$$

$$a(b+c) = ab+ac$$

48.



Given: $\triangle ACD$ with $BE \parallel CD$
 Prove: $\triangle ABE \sim \triangle ACD$

PROOF

1. $BE \parallel CD$

Given

2. $\angle ABE = \angle C$
 $\angle AEB = \angle D$

Parallel Lines form equal corresponding angles

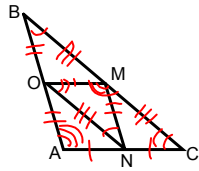
(2.5 $\angle A = \angle A$)

Reflexive Property

3. $\triangle ABE \sim \triangle ACD$

AA Similarity

50.



Given: $\triangle ABC$ with midsegments MN , MO , and NO

Prove: $\triangle MNO \sim \triangle ABC$

PROOF

1. MN, MO, NO are midsegments Given

2. $AN = NC$, $BO = OA$,
 $BM = MC$

Midsegments connect midpoints O, M, N
midpoint \Rightarrow divide segments into 2 equal segments

Midsegment Theorem

3. $OM \parallel AC$,
 $MN \parallel BA$,
 $ON \parallel BC$

$OM = AN = NC$

$MN = BO = OA$

$ON = BM = MC$

4. $\triangle BOM \cong \triangle OAN$
 $\cong \triangle MNC$
 $\cong \triangle NMO$

SSS
congruence

5. $\angle B = \angle MNO$

$\angle A = \angle NMO$

corresponding parts
of $\cong \triangle$'s are $=$

6. $\triangle ABC \sim \triangle MNO$ AA similarity