HW #2 - Due Wed. 3/15

11.1
- set I p. 431 #15,17,18 (use Theorem 49 & its corollaries for find x)
- set II p. 432 #48-51 (SAT problem)
- set III p. 433 #2-8 (Golden Ratio)

11.2
- set I p. 437 #18-25 (Pythagorean triples)
- set II p. 439 #47-50 (oil well puzzle)
- set III p. 440 #1-3 (Fermat's Last Theorem)

11.3
- set I p. 445 #30-33 (St. Peter's Cathedral & Olympic sailing competition)

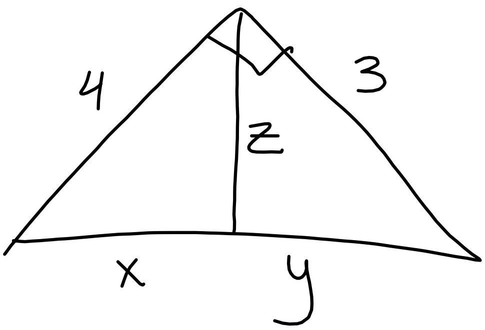
11.4
- set I p. 450 #17-28 (tangent ratios in 30-60-90 and 3-4-5 triangles)

11.5
- set I p. 456 #13-22 (sine and cosine ratios in 5-12-13 triangle
- set II p 460 #56-60 (dominos)
- set III p. 460 #1-2 (Proxima Centauri)

HW #3 - Due Fri. 3/17

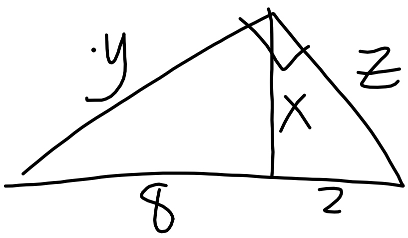
11.6
- set I p. 463 #1-8; 23-30 (slope & distance)
- set II p. 466 #52-65 (proofs w/ parallel, perpendicular lines)

11.7
- set I p. 471-472 #12-17 (law of sines, law of cosines)



$$x+y=5$$

$$\frac{x}{4}=\frac{4}{5} \Rightarrow x=\frac{16}{5}$$



$$\frac{2}{x}=\frac{x}{8}$$

$$\frac{8}{y}=\frac{y}{8+2}$$

$$\frac{2}{z}=\frac{z}{8+2}$$

Theorem 49: The altitude to the hypotenuse of a right triangle forms two triangles similar to it and to each other.

Corollary 1 to Theorem 49:
The altitude to the hypotenuse of a right triangle is the geometric mean between the segments into which it divides the hypotenuse.

Corollary 2 to Theorem 49:
Each leg of a right triangle is the geometric mean between the hypotenuse and its projection on the hypotenuse.

Pythagorean Theorem: In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs.

Theorem 50 – The Isosceles Right Triangle Theorem
In an isosceles right triangle, the hypotenuse is $\sqrt{2}$ times the length of a leg.

Corollary to Theorem 50 – Each diagonal of a square is $\sqrt{2}$ times the length of one side.

Theorem 51 – The $30° - 60°$ Right Triangle Theorem
In a $30° - 60°$ right triangle, the hypotenuse is twice the shorter leg and the longer leg is $\sqrt{3}$ times the shorter leg.

Corollary to Theorem 51
An altitude of an equilateral triangle having side s is $\frac{\sqrt{3}}{2}s$ and its area is $\frac{\sqrt{3}}{4}s^2$.

11.4 – The Tangent Ratio

Def: The tangent of an acute angle of a right triangle is the ratio of the length of the opposite leg to the length of the adjacent leg.

11.5 – The Sine and Cosine Ratios

Def: The sine of an acute angle of a right triangle is the ratio of the length of the opposite leg to the length of the hypotenuse.

Def: The cosine of an acute angle of a right triangle is the ratio of the length of the adjacent leg to the length of the hypotenuse.

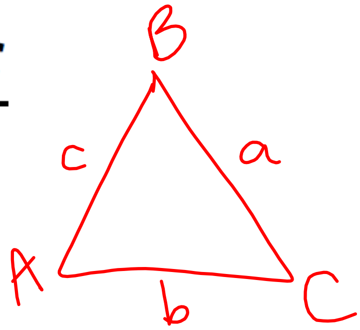
11.6 – Slope

Theorem 52 – Two nonvertical lines are parallel iff their slopes are equal.

Theorem 53 – Two nonvertical lines are perpendicular iff the product of their slopes is -1.

The Law of Sines

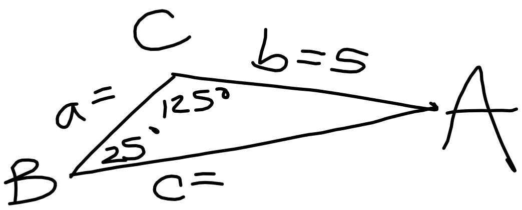
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



or

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$B = 25°$, $C = 125°$, $b = 5$



$A = 180 - 125 - 25$

$A = 30°$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{a}{\sin 30°} = \frac{5}{\sin 25°}$$

$$a = \frac{5 \sin 30°}{\sin 25°} \approx \boxed{5.9}$$

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

$$\frac{c}{\sin 125°} = \frac{5}{\sin 25°}$$

$$c = \frac{5 \cdot \sin 125°}{\sin 25°} \approx \boxed{9.7}$$

$B = 54.8^\circ,\ C = 72.6^\circ,\ a = 14.4$

Find side c (17.3) & side b (14.8) using Law of Sines.

$A = 180^\circ - 54.8^\circ - 72.6^\circ = 52.6^\circ$

$$\frac{b}{\sin B} = \frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{b}{\sin 54.8^\circ} = \frac{14.4}{\sin 52.6^\circ}$$

$$b = \frac{14.4 \sin 54.8^\circ}{\sin 52.6^\circ} \approx 14.8$$

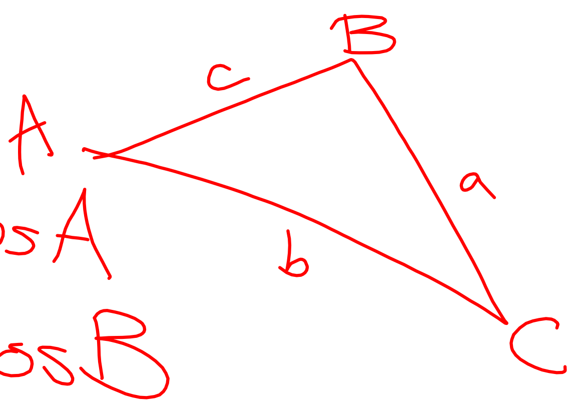
$$\frac{c}{\sin 72.6^\circ} = \frac{14.4}{\sin 52.6^\circ}$$

$$c = \frac{14.4 \sin 72.6^\circ}{\sin 52.6^\circ} \approx 17.3$$

<u>The Law of Cosines</u>

SSS

SAS



$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cdot \cos C$$

16. $a = 60, b = 88, c = 120.\ B = ?$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$

$$\frac{\cancel{2ac} \cdot \cos B}{\cancel{2ac}} = \frac{a^2 + c^2 - b^2}{2ac}$$

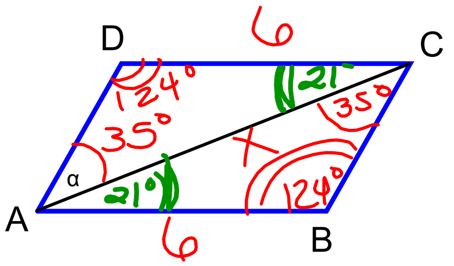
$$\cos^{-1}(\cos B) = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right)$$

$$B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right)$$

$$= \cos^{-1}\left(\frac{(60^2 + 120^2 - 88^2)}{(2(60)(120))}\right)$$

$$\approx 44.6^\circ$$

The longer side of a parallelogram is 6.0 meters. The measure of angle BAD is 56º and α is 35º. Find the length of the longer diagonal.



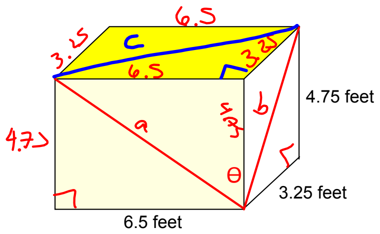
$m\angle BAD = 56°$

$$\angle B = \angle D = 180° - 56° = 124°$$

$$\frac{x}{\sin 124°} = \frac{6}{\sin 35°}$$

$$x = \frac{6 \sin 124°}{\sin 35°} \approx \boxed{8.7}$$

The rectangular box in the figure measures 6.50 feet by 3.25 feet by 4.75 feet. Find the measure of the angle Θ that is formed by the union of the diagonal shown on the front of the box and the diagonal shown on the right side of the box.



$a^2 = 6.5^2 + 4.75^2$

$a = \sqrt{6.5^2 + 4.75^2} \approx 8.05$

$b^2 = 3.25^2 + 4.75^2$

$b = \sqrt{3.25^2 + 4.75^2} \approx 5.76$

$c^2 = 6.5^2 + 3.25^2$

$c = \sqrt{6.5^2 + 3.25^2} \approx 7.27$

$c^2 = a^2 + b^2 - 2ab \cdot \cos\Theta$

$2ab\cos\Theta = a^2 + b^2 - c^2$

$\cos\Theta = \dfrac{a^2 + b^2 - c^2}{2ab}$

$\Theta = \cos^{-1}\left(\dfrac{a^2 + b^2 - c^2}{2ab}\right) = \cos^{-1}\left(\dfrac{8.05^2 + 5.76^2 - 7.27^2}{2(8.05)(5.76)}\right)$

$\approx \boxed{60.9^\circ}$