Test Monday -

Heavy emphasis on vocab/theorems!

Ch 13 - The Concurrence Theorems

13.1 - Triangles and Circles

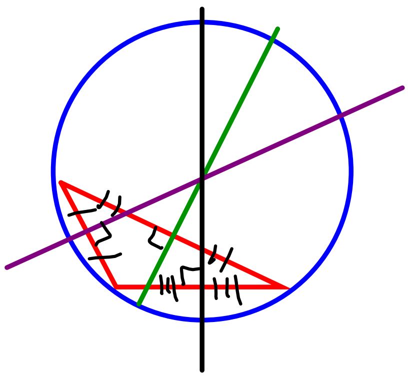
Def: A polygon is **cyclic** iff there exists a circle that contains all of its vertices.

Theorem 68: Every Triangle is cyclic.

Def: A polygon is **inscribed in a circle** iff each vertex of the polygon lies on the circle. The circle is **circumscribed about the polygon**.

Corollary to Theorem 68: The perpendicular bisectors of the sides of a triangle are concurrent.

Construction 9: To circumscribe a circle about a triangle.

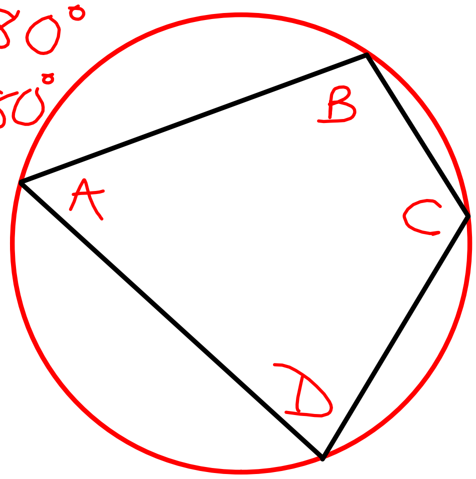


13.2 - Cyclic Quadrilaterals

Theorem 69: A quadrilateral is cyclic iff a pair of its opposite angles are supplementary.

$\angle A + \angle C = 180^\circ$

$\angle B + \angle D = 180^\circ$



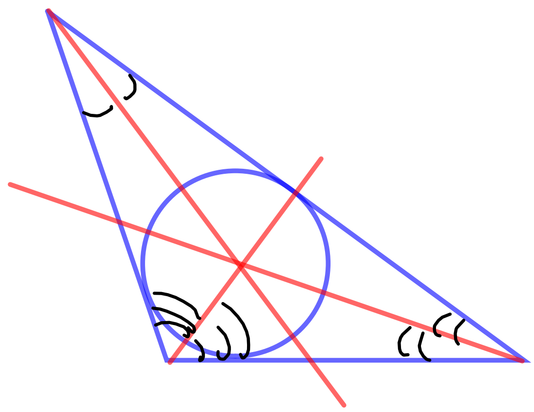
13.3 - Incircles

Def: A circle is **inscribed in a polygon** iff each side of the polygon is tangent to the circle. The polygon is **circumscribed about the circle**. The circle is called the **incircle** of the polygon and its center is called the **incenter** of the polygon.

Theorem 70: Every triangle has an incircle.

Corollary to Theorem 70: The angle bisectors of a triangle are concurrent.

Construction 10: To inscribe a circle in a triangle.

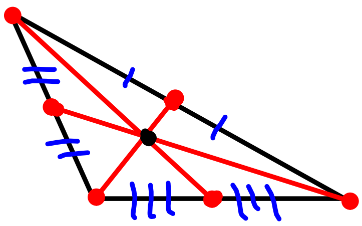


13.4 - The Centroid of a Triangle

Def: A **median** of a triangle is a line segment that joins a vertex to the midpoint of the opposite side.

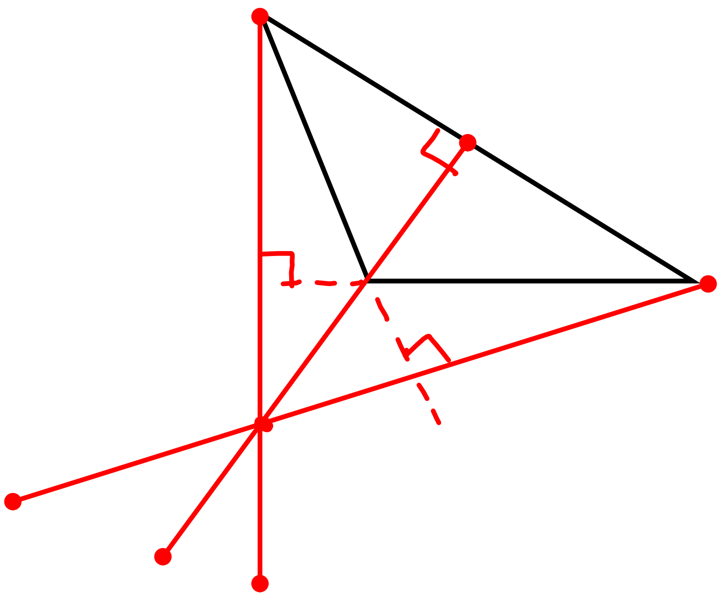
Theorem 71: The medians of a triangle are concurrent.

Def: The **centroid** of a triangle is the point in which its medians are concurrent.



Theorem 72: The lines containing the altitudes of a triangle are concurrent.

Def: The **orthocenter** of a triangle is the point in which the lines containing its altitudes are concurrent.



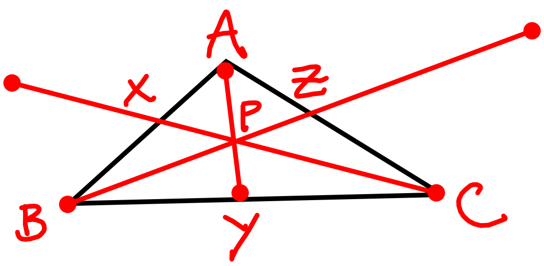
13.5 - Ceva's Theorem

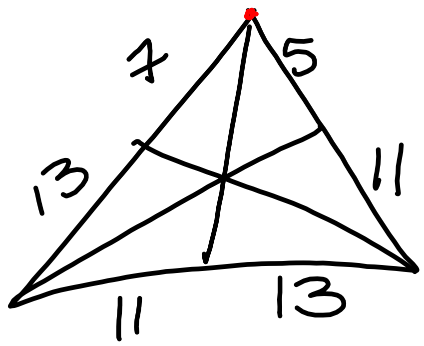
Def: A cevian of a triangle is a line segment that joins a vertex of the triangle to a point on the opposite side.

Theorem 73: Ceva's Theorem

Three cevians, AY, BZ, and CX of ΔABC are concurrent iff

$$\frac{AX}{XB}\cdot\frac{BY}{YC}\cdot\frac{CZ}{ZA}=1$$

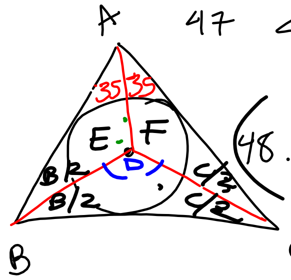




Are these cevians concurrent?

$$\frac{7}{13}\cdot\frac{11}{13}\cdot\frac{11}{5}\neq 1$$

NO



47 $\angle A = 70^\circ$, $\angle ABC = 60^\circ$

$\angle D = ?$ 125°

(48. $\angle A = 70$, $\angle ABC = 40^\circ$)

$\angle D = ?$

49. $\angle A = 70^\circ$, $\angle ABC$ unknown

$\angle D = ?$

$\angle E = 180^\circ - 35 - B/2$

$\angle F = 180^\circ - 35 - C/2$

$\angle D = 180^\circ - B/2 - C/2$

$\angle D = 360 - \angle E - \angle F$

$= \cancel{360} - (\cancel{180} - 35 - B/2) - (\cancel{180} - 35 - C/2)$

$= 70 + B/2 + C/2$

$\angle A + 90 + \frac{\angle A}{2}$

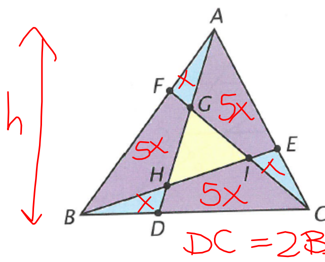
$= 90 + \frac{3\angle A}{2}$

$\angle A + \angle B + \angle C = 180$

$\frac{\angle B + \angle C}{2} = \frac{180 - \angle A}{2}$

$\frac{\angle B}{2} + \frac{\angle C}{2} = 90 - \frac{\angle A}{2}$

Area Puzzle. Rather than being concurrent, the three cevians in $\triangle ABC$ intersect to form a smaller triangle, $\triangle GHI$.



DC = 2BD

Given that D, E, and F are points of trisection of the sides of $\triangle ABC$, it can be proved that the blue regions have equal areas and the purple regions have equal areas; it can also be proved that, if the area of a blue region is x, the area of a purple region is $5x$.

Find the areas of the following triangles in terms of x.

1. $\triangle ABD$. 2. $\triangle ADC$. 3. $\triangle GHI$.
4. What fraction of the area of $\triangle ABC$ is the area of $\triangle GHI$?

1. area of $\triangle ABD$

$x + 5x + x = \boxed{7x} = \frac{1}{2} BD \cdot h$

2. area of $\triangle ADC$

$= \frac{1}{2} DC \cdot h = \frac{1}{2}(2BD)h = 2\left(\frac{1}{2} BD\, h\right)$

$= 2(7x) = \boxed{14x}$

3. area of $\triangle GHI$

$= \text{area } \triangle ADC - (5x + x + 5x)$

$= 14x - 11x = \boxed{3x}$

4. $\frac{\text{area of } \triangle GHI}{\text{area of } \triangle ABC} = \frac{3x}{21x} = \boxed{\frac{1}{7}}$

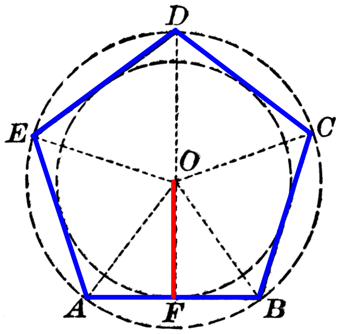
Ch 14 - Regular Polygons and the Circle

14.1 - Regular Polygons

Def: A **regular polygon** is a convex polygon that is both equilateral and equiangular.

Theorem 74: Every regular polygon is cyclic.

Def: An **apothem** of a regular polygon is a perpendicular line segment from its center to one of its sides.

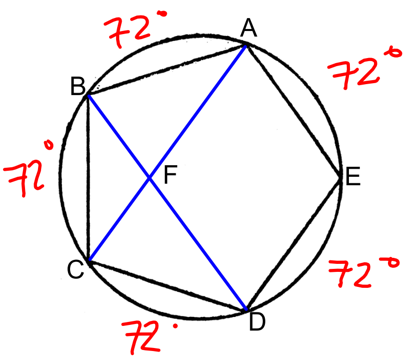


O is the center of pentagon ABCDE

OF is an apothem

OB is a radius of the pentagon (segment connecting the center to a vertex)

BOC is a central angle



52. Why are the five arcs into which the pentagon divides the circle equal?

equal chords have equal arcs

What is the measure of...

53. each arc

$\frac{360}{5} = 72°$

54. angle ABD

$\frac{1}{2}(72° + 72°) = 72°$

55. angle AFB

$\frac{1}{2}(72° + 72°) = 72°$

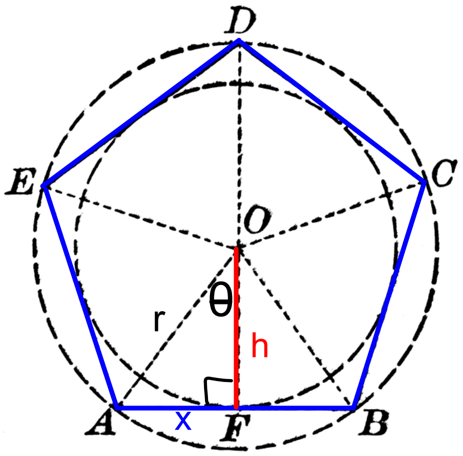
56. angle BAC

$\frac{1}{2}(72°) = 36°$

57. angle BCA

$\frac{1}{2}(72°) = 36°$

58. $m\angle ABC = 180° - (36° + 36°) = \boxed{108°}$



What is the perimeter of a regular n-gon?

$\theta = \frac{360°}{2n} = \frac{180°}{n}$

$\sin\theta = \frac{x}{r}$; ~~$\cos\theta = \frac{h}{r}$~~

$x = r\sin\theta$

$x = r\sin\frac{180°}{n}$

single side length of n-gon $= 2r\sin\frac{180°}{n}$

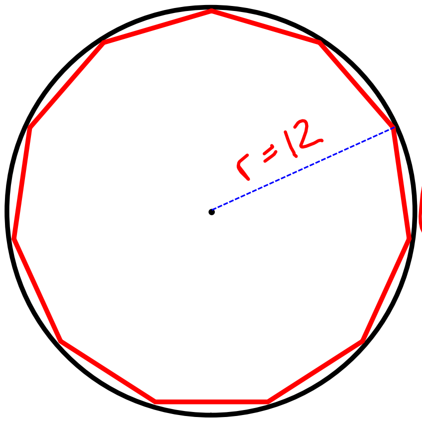
total perimeter $= 2nr\sin\frac{180°}{n}$

14.2 – The Perimeter of a Regular Polygon

Theorem 75 – The perimeter of a regular polygon having n sides is $2Nr$, in which $N = n\sin\frac{180}{n}$ and r is its radius.

Length of one side of a regular n-gon is $2r\sin\frac{180}{n}$

Perimeter of a regular n-gon is $2nr\sin\frac{180}{n}$



Regular 11-gon of radius 12

Circumference of circle?

$C = 2\pi r = 2\pi(12) = 24\pi$

≈ 75.40

Perimeter of 11-gon?

$P = 2nr\sin\frac{180^\circ}{n} = 2(11)(12)\sin\frac{180^\circ}{11}$

≈ 74.38