4.2 Solving systems of linear equations using the elimination method and matrices

42. $2x-y+2z=7$ (1)

$x+y+z=2$ (2)

$3x-y+z=6$ (3)

1 + 2

$2x-y+2z=7$

$x+y+z=2$

$3x+3z=9$

2 + 3

$x+y+z=2$

$3x-y+z=6$

$4x+2z=8$

$\begin{cases} (3x+3z=9)\cdot 2 \\ (4x+2z=8)\cdot(-3) \end{cases} \Rightarrow \begin{cases} 6x+6z=18 \\ -12x-6z=-24 \end{cases}$

$-6x = -6$

$x=1$

$3(1)+3z=9$

$3z=6$

$z=2$

$1+y+2=2$

$y=-1$

$(1,-1,2)$

50. $2x+4y-2z=3$ (1)
$x+3y+4z=1$ (2)
$x+2y-z=4$ (3)

1 & 2

$\begin{cases} 2x+4y-2z=3 \\ (x+3y+4z=1)(-2) \end{cases}$

$\begin{cases} 2x+4y-2z=3 \\ -2x-6y-8z=-2 \end{cases}$

$-2y-10z=1$

3 & 2

$\begin{cases} (x+2y-z=4)(-1) \\ x+3y+4z=1 \end{cases}$

$\begin{cases} -x-2y+z=-4 \\ x+3y+4z=1 \end{cases}$

$y+5z=-3$

$\begin{cases} -2y-10z=1 \\ (y+5z=-3)\cdot(2) \end{cases} \Rightarrow \begin{cases} -2y-10z=1 \\ 2y+10z=-6 \end{cases}$

$0=-5$

no solution

$y=mx+b$

$(x, mx+b)$

$\left(\frac{y-b}{m}, y\right)$

$-2y-10z=1$
$2y+10z=-1$
$0=0$

$2y=-10z-1$
$y=-5z-\frac{1}{2}$

$\left(11z+\frac{5}{2}, -5z-\frac{1}{2}, z\right)$

$x+3y+4z=1$
$x+3\left(-5z-\frac{1}{2}\right)+4z=1$
$x-15z-\frac{3}{2}+4z=1$
$x=11z+\frac{5}{2}$

$$\begin{cases} ax+by=c \\ dx+ey=f \end{cases} \Rightarrow \left[\begin{array}{cc|c} a & b & c \\ d & e & f \end{array}\right]$$

rewrite ⋯⋯> $\left[\begin{array}{cc|c} 1 & 0 & x \\ 0 & 1 & y \end{array}\right]$

if 3 var's $\left[\begin{array}{ccc|c} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z \end{array}\right]$

6. $\begin{cases} x - 3y = 4 \\ x + 5y = -4 \end{cases} \Rightarrow \left[\begin{array}{cc|c} 1 & -3 & 4 \\ 1 & 5 & -4 \end{array}\right]$

$\xrightarrow{R2-R1} \left[\begin{array}{cc|c} 1 & -3 & 4 \\ 0 & 8 & -8 \end{array}\right] \xrightarrow{\frac{1}{8}\cdot R2} \left[\begin{array}{cc|c} 1 & -3 & 4 \\ 0 & 1 & -1 \end{array}\right]$

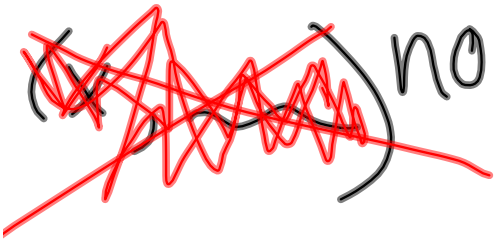
$\xrightarrow{R1+3\cdot R2} \left[\begin{array}{cc|c} 1+3\cdot 0 & -3+3\cdot 1 & 4+3(-1) \\ 0 & 1 & -1 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -1 \end{array}\right]$

$(1, -1)$

14. $\begin{cases} 3x+6y=7 \\ 2x+4y=5 \end{cases} \Rightarrow \left[\begin{array}{cc|c} 3 & 6 & 7 \\ 2 & 4 & 5 \end{array}\right]$

R1 - R2

$\left[\begin{array}{cc|c} 1 & 2 & 2 \\ 2 & 4 & 5 \end{array}\right]$ R2 - 2·R1 $\longrightarrow \left[\begin{array}{cc|c} 1 & 2 & 2 \\ 0 & 0 & 1 \end{array}\right]$ $x+2y=2$, $0=1$



no solution!

24. $\begin{cases} 4x - 8y = 5 \\ 8x + 2y = 1 \end{cases} \Rightarrow \left[\begin{array}{cc|c} 4 & -8 & 5 \\ 8 & 2 & 1 \end{array}\right]$

$R1 \cdot \frac{1}{4}$ $\quad R_2 - 8 \cdot R1$ $\quad 1 - 8 \cdot \frac{5}{4}$

$\left[\begin{array}{cc|c} 1 & -2 & 5/4 \\ 8 & 2 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & -2 & 5/4 \\ 0 & 18 & -9 \end{array}\right] \xrightarrow{R2 \cdot \frac{1}{18}} \left[\begin{array}{cc|c} 1 & -2 & 5/4 \\ 0 & 1 & -\frac{1}{2} \end{array}\right]$

$2 - 8(-2)$ $\quad 2 + 16$ $\quad \frac{5}{4} + 2\left(\frac{-1}{2}\right) = \frac{5}{4} - \frac{4}{4} = \frac{1}{4}$

$\left[\begin{array}{cc|c} 1 & -2 & 5/4 \\ 0 & 1 & -1/2 \end{array}\right] \xrightarrow{R1 + 2R2} \left[\begin{array}{cc|c} 1 & 0 & 1/4 \\ 0 & 1 & -1/2 \end{array}\right]$ $\quad \left(\frac{1}{4}, -\frac{1}{2}\right)$

HW
4.2
#41-51 odd