4.2 Solving systems of linear equations using the elimination method and matrices

42. $2x - y + 2z = 7$ (1)
$x + y + z = 2$ (2)
$3x - y + z = 6$ (3)

1 & 2
$$\begin{array}{r} 2x - y + 2z = 7 \\ x + y + z = 2 \\ \hline 3x \quad + 3z = 9 \end{array}$$

2 & 3
$$\begin{array}{r} x + y + z = 2 \\ 3x - y + z = 6 \\ \hline 4x \quad + 2z = 8 \end{array}$$

$$\begin{cases} (3x + 3z = 9) \cdot 2 \\ (4x + 2z = 8) \cdot (-3) \end{cases} \Rightarrow \begin{cases} 6x + 6z = 18 \\ -12x - 6z = -24 \end{cases}$$

$$\begin{array}{r} 6x + 6z = 18 \\ -12x - 6z = -24 \\ \hline -6x \quad = -6 \end{array}$$

$x = 1$

$3(1) + 3z = 9$
$3z = 6$
$z = 2$

$1 + y + 2 = 2$
$y = -1$

$\boxed{(1, -1, 2)}$

50. $2x+4y-2z=3$ (1)
$x+3y+4z=1$ (2)
$x+2y-z=4$ (3)

1 & 2

$2x+4y-2z=3$
$(x+3y+4z=1)\cdot(-2)$

$$\begin{cases} 2x+4y-2z=3 \\ -2x-6y-8z=-2 \end{cases}$$
$-2y-10z=1$

2 & 3

$(x+3y+4z=1)(\cdot -1)$
$x+2y-z=4$

$$\begin{cases} -x-3y-4z=-1 \\ x+2y-z=4 \end{cases}$$
$-y-5z=3$

$$\begin{cases} -2y-10z=1 \\ (-y-5z=3)\cdot(-2) \end{cases} \Rightarrow \begin{cases} -2y-10z=1 \\ 2y+10z=-6 \end{cases}$$
$0=-5$

no solution!

$0=0 \Rightarrow$ dependent system

$(x, mx+b)$

$$\begin{cases} ax + by = c \\ dx + ey = f \end{cases} \Rightarrow \left[\begin{array}{cc|c} a & b & c \\ d & e & f \end{array}\right]$$

rewrite $\longrightarrow$ $\left[\begin{array}{cc|c} 1 & 0 & x \\ 0 & 1 & y \end{array}\right]$

if 3 var's

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z \end{array}\right]$$

6. $\begin{cases} x - 3y = 4 \\ x + 5y = -4 \end{cases} \Rightarrow \left[\begin{array}{cc|c} 1 & -3 & 4 \\ 1 & 5 & -4 \end{array}\right]$

$\xrightarrow{R2+(-1)R1} \left[\begin{array}{cc|c} 1 & -3 & 4 \\ 0 & 8 & -8 \end{array}\right]$

$0+(-1)(1)$ $5+(-1)(-3)$ $-4+(-1)(4)$

$\xrightarrow{R_2 \cdot \frac{1}{8}} \left[\begin{array}{cc|c} 1 & -3 & 4 \\ 0 & 1 & -1 \end{array}\right]$

$\xrightarrow{R1+(3)R2} \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -1 \end{array}\right]$

$\boxed{(1,-1)}$

14. $\begin{cases} 3x + 6y = 7 \\ 2x + 4y = 5 \end{cases} \Rightarrow \left[\begin{array}{cc|c} 3 & 6 & 7 \\ 2 & 4 & 5 \end{array}\right]$

$\xrightarrow{R1 - R2} \left[\begin{array}{cc|c} 1 & 2 & 2 \\ 2 & 4 & 5 \end{array}\right] \xrightarrow{R2 + (-2)R1} \left[\begin{array}{cc|c} 1 & 2 & 2 \\ 0 & 0 & 1 \end{array}\right]$

$2+(-2)(1),\ 4+(-2)(2),\ 5+(-2)(2)$

<u>no solution</u>

$x + 2y = 2$

$0 = 1$

$\left[\begin{array}{cc|c} 1 & 2 & 2 \\ 0 & 0 & 0 \end{array}\right]$

$x + 2y = 2$

$0 = 0$

$2y = -x + 2$

$y = -\frac{1}{2}x + 1$

dependent

$(x, mx+b)$

$(x, -\frac{1}{2}x + 1)$

24. $\begin{cases} 4x-8y=5 \\ 8x+2y=1 \end{cases} \Rightarrow \left[\begin{array}{cc|c} 4 & -8 & 5 \\ 8 & 2 & 1 \end{array}\right]$

$2-8(-2)$
$1-8\left(\frac{5}{\cancel{4}_{2}}\right)$

$\xrightarrow{R1 \cdot \frac{1}{4}} \left[\begin{array}{cc|c} 1 & -2 & 5/4 \\ 8 & 2 & 1 \end{array}\right] \xrightarrow{R_2 - 8 \cdot R1} \left[\begin{array}{cc|c} 1 & -2 & 5/4 \\ 0 & 18 & -9 \end{array}\right]$

$\xrightarrow{R2 \cdot 1/18} \left[\begin{array}{cc|c} 1 & -2 & 5/4 \\ 0 & 1 & -1/2 \end{array}\right] \xrightarrow{R_1 + 2R_2} \left[\begin{array}{cc|c} 1 & 0 & 1/4 \\ 0 & 1 & -1/2 \end{array}\right]$

$\frac{5}{4} + \frac{\cancel{2}}{1}\left(\frac{-1}{\cancel{2}}\right) = \frac{5}{4} - 1 \cdot \frac{4}{4} = \frac{1}{4}$

$\boxed{\left(1/4, -1/2\right)}$

HW

4.2

#41-51 odd

solve by elimination method