5.1

81. $\left(\frac{9ab^{-2}}{8a^{-2}b}\right)^{-2}\left(\frac{3a^{-2}b}{2a^{2}b^{-2}}\right)^{3}$

$= \frac{3^{-4}a^{-2}b^{4}}{2^{-6}a^{4}b^{-2}} \cdot \frac{3^{3}a^{-6}b^{3}}{2^{3}a^{6}b^{-6}}$

$= \frac{2^{3}a^{-2+(-6)-4-6}b^{4+3-(-2)-(-6)}}{3} = \frac{8b^{15}}{3a^{18}}$

77. $\dfrac{x^{2n-1}\,y^{n-3}}{x^{n+4}\,y^{n+3}} = x^{2n-1-(n+4)}\,y^{n-3-(n+3)}$

$= x^{n-5}\,y^{-6} = \dfrac{x^{n-5}}{y^{6}}$

5.1 Scientific Notation

$3.14 \times 10^{5} = 314000$

$5.7 \times 10^{-9} = 0.0000000057$

$\square.\sim \times 10^{\text{integer}}$

↑ single digit 1-9

$5,243,000 = 5.243 \times 10^{6}$

$0.0073 = 7.3 \times 10^{-3}$

5.2 Introduction to Polynomials

A polynomial is an expression consisting of variables and constants, using only the operations of addition, subtraction, multiplication, and non-negative integer exponents.

A polynomial with one term is a monomial.

e.g. $3x^2$ or $5xy^3$

A polynomial with two terms is a binomial.

e.g. $7xy^5 - 3x$ or $xyzw + 23x^2$ or $x - 2$

A polynomial with three terms is a trinomial.

e.g. $x^2 + 5x - 6$

The degree of a monomial is the sum of the exponents of the variables.

$7x^1y^5$ has degree 6

$xyzw$ has degree 4

$13x^3yz^2$ has degree 6

$-2ab^3$ has degree 4

The degree of a polynomial is the greatest of the degrees of any of its terms.

$x^2 + 5x - 6$ has degree 2

$3xy - 15x^3y + 2v^3xz$ has degree 5

$15xy^2 - \sqrt{2}x + 32xyz - 5000$ has degree 3

The terms of a polynomial in only one variable are usually arranged in descending order, so that the exponents of the variable decrease from left to right, in the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_2 x^2 + a_1 x + a_0 x^0$$

$a_n, \ldots, a_0$ are real-numbered coefficients

$a_n x^n$ is the lead term (term containing the variable with the largest exponent)

a_n is the leading coefficient (coefficient of the variable with the largest exponent)

a_0 is the constant term (term without a variable)

n is the degree of the polynomial (largest exponent)

The linear function $f(x) = mx + b$ is a polynomial of degree one.

A second-degree polynomial of the form $f(x) = ax^2 + bx + c$ is called a quadratic function.

A third-degree polynomial is called a cubic function.

Problems from Section 5.2:

Is it a polynomial? If so, state the lead term, leading coefficient, degree, and constant term.

16. $P(x) = 3x^4 - 3x - 7$

Lead term: $3x^4$
Leading coefficient: 3
Degree: 4
Constant term: -7

18. $R(x) = \frac{3x^2-2x+1}{x}$

Lead term:
Leading coefficient:
Degree:
Constant term:

not a polynomial (no variables in denominator)

20. $f(x) = x^2 - \sqrt{x+2} - 8$

Lead term:
Leading coefficient:
Degree:
Constant term:

not a polynomial (no variables under radical!)

22. $g(x) = -4x^5 + 3x^2 + x - \sqrt{7}$

Lead term: $-4x^5$
Leading coefficient: -4
Degree: 5
Constant term: $-\sqrt{7}$

To evaluate a polynomial, replace the variable by its value and simplify.

6. Given $R(x) = -x^3 + 2x^2 - 3x + 4$, evaluate $R(-1)$.

$R(-1) = -(-1)^3 + 2(-1)^2 - 3(-1) + 4 =$

$= 1 + 2 + 3 + 4 =$

$= 10$

$-1^2 = -1$

$(-1)^2 = 1$

Polynomials can be added by combining like terms.

36. $(3x^2 - 2x + 7) + (-3x^2 + 2x - 12)$

$X^2 + X \neq X^3$

$3x^2 - 3x^2 - 2x + 2x + 7 - 12$

$= -5$

42. $(3a^2 - 9a) - (-5a^2 + 7a - 6)$

$(3a^2 - 9a) + (-1)(-5a^2 + 7a - 6)$

$3a^2 - 9a + 5a^2 - 7a + 6$

$8a^2 - 16a + 6$

50. $(2x^4 - 2x^2 + 1) - (3x^3 - 2x^2 + 3x + 8)$

$2x^4 - 3x^3 - 3x - 7$

46. $(2x^{2n} - x^n - 1) - (5x^{2n} + 7x^n + 1)$

$-3x^{2n} - 8x^n - 2$

Homework:
5.2 #3-7odd, 15-25odd, 35-49odd