Review Problems:

1. Write in interval notation: $\{x|x \leq -5\}$

$(-\infty, -5]$

2. Solve. Write the solution set in interval notation.

$6 - 4x > -3$ and $3x + 2 \leq -1$

$-4x > -9$ $\quad 3x \leq -3$

$x < 9/4$ $\quad x \leq -1$ $\quad (-\infty, -1]$

3. Simplify. $(x^5x^{-3})^{-2}(x^{-4}x^3)^4$

$(x^2)^{-2}(x^{-1})^4$

$x^{-4}x^{-4}$

$x^{-8} = \frac{1}{x^8}$

Homework Quesons?

5.3

25. $a^{n+1}(a^n - 3a^1 + 2)$

$= a^{2n+1} - 3^{n+2} + 2a^{n+1}$

65. $(b-3)(3b-2)(b-1)$

$(3b^2 - 2b - 9b + 6)(b-1)$

$(3b^2 - 11b + 6)(b-1)$

$3b^3 - 11b^2 + 6b - 3b^2 + 11b - 6$

$3b^3 - 14b^2 + 17b - 6$

67. $(x^n-y^n)(x^{2n}-3x^ny^n-y^{2n})$

$x^{3n}-3x^{2n}y^n-x^ny^{2n}-x^{2n}y^n+3x^{n\cdot 2n}y^{3n}+y^{3n}$

5.4 Division of Polynomials

Long Division $57\div 2$

$$2\overline{)57}$$

28

-4

17

-16

1

$57=2\cdot 28+1$

4. $(x^2 - 14x + 24) \div (x - 2)$

$x - 12$

$x-2 \overline{)\, x^2 - 14x + 24}$

$-(x^2 - 2x)$

$-12x + 24$

$-(-12x + 24)$

0

Quotient: $x-12$

Remainder: 0

$x^2 - 14x + 24 = (x-2)(x-12) + 0$

6. $(x^3 + 4x^2 - 8) \div (x + 4)$

x^2

$x+4 \overline{)\, x^3 + 4x^2 - 8}$

$-(x^3 + 4x^2)$

-8

Quotient: x^2

Remainder: -8

$x^3 + 4x^2 - 8 = (x+4)(x^2) - 8$

10. $(18x^2 - 3x + 2) \div (3x + 2)$

$6x - 5$

$3x+2 \overline{)\, 18x^2 - 3x + 2}$

$-(18x^2 + 12x)$

$-15x + 2$

$-(-15x - 10)$

12

Quotient: $6x-5$

Remainder: 12

14. $(12x^4 - 11x^2 + 10) \div (3x^2 + 1)$

$4x^2 - 5$

$3x^2+1 \overline{)\, 12x^4 - 11x^2 + 10}$

$-(12x^4 + 4x^2)$

$-15x^2 + 10$

$-(-15x^2 - 5)$

15

Q: $4x^2 - 5$

R: 15

20. $\frac{x+3x^4-x^2+5x^3-2}{x+2}$ $= 3x^3-x^2+x-1$

$3x^3-x^2+x-1$

$x+2 \overline{)\ 3x^4+5x^3-x^2+x-2}$

$-(3x^4+6x^3)$

$-x^3-x^2+x-2$

$-(-x^3-2x^2)$

x^2+x-2

$-(x^2+2x)$

$-x-2$

$-(-x-2)$

0

24. $\frac{2-3x^2+5x^3}{x^2+3}$

$5x-3$

$x^2+3 \overline{)\ 5x^3-3x^2+2}$ Q: $5x-3$

$-(5x^3+15x)$

$-3x^2-15x+2$ R: $-15x+1$

$-(-3x^2-9)$

$-15x+11$

$5x^3-3x^2+2 = (x^2+3)(5x-3)+(-15x+11)$

$\frac{5x^3-3x^2+2}{} = 5x-3+\frac{-15x+11}{}$

Synthec Division

only works when dividing by $(x \pm a)$

28. $(3x^2+19x+20) \div (x+5)$ $\frac{x^2}{x} = x$

Q: $3x+4$

-5	3	19	20
		$-5 \cdot 3$	$-5 \cdot 4$
		-15	-20
	3	$19+(-15)$	0
	linear coeff.	constant	remainder

30. $(4x^2-8) \div (x-2)$

missing x-term

2	4	0	-8
		8	16
	4	8	8

R: 8

Q: $4x+8$

34. $(3x^2 - 15) \div (x + 3)$

$-3 \mid 3 \quad 0 \quad -15$

$\quad\quad -9 \quad 27$

$3 \quad -9 \mid 12$

Q: $3x - 9$

R: 12

38. $(x^3 - 4x^2 + x + 6) \div (x + 1)$

$-1 \mid 1 \quad -4 \quad 1 \quad 6$

$\quad\quad -1 \quad 5 \quad -6$

$1 \quad -5 \quad 6 \mid 0$

Q: $x^2 - 5x + 6$

R: 0

$x^3 - 4x^2 + x + 6 = (x+1)(x^2 - 5x + 6)$

42. $(x^3 + 2x + 5) \div (x - 2)$

$2 \mid 1 \quad 0 \quad 2 \quad 5$

$\quad\quad 2 \quad 4 \quad 12$

$1 \quad 2 \quad 6 \mid 17$

R: 17

Q: $x^2 + 2x + 6$

48. $\dfrac{3 - 13x - 5x^2 + 9x^3 - 2x^4}{3 - x}$

52. $\dfrac{x^4 - 3x^3 - 30}{x + 2}$

$$\begin{array}{r|rrrrr} -2 & 1 & -3 & 0 & 0 & -30 \\ & & -2 & 10 & -20 & 40 \\ \hline & 1 & -5 & 10 & -20 & \boxed{10} \end{array}$$

Q: $x^3 - 5x^2 + 10x - 20$

R: 10

Evaluate a Polynomial using Synthetic Division

Remainder Theorem: If the polynomial $P(x)$ is divided by $x - a$, the remainder is $P(a)$.

56. $Q(x) = 3x^2 - 5x - 1;\ Q(2) = \boxed{1}$

$$\begin{array}{r|rrr} 2 & 3 & -5 & -1 \\ & & 6 & 2 \\ \hline & 3 & 1 & \boxed{1} \end{array}$$

60. $R(t) = 3t^3 + t^2 - 4t + 2;\ R(-3) = \boxed{-58}$

$$\begin{array}{r|rrrr} -3 & 3 & 1 & -4 & 2 \\ & & -9 & 24 & -60 \\ \hline & 3 & -8 & 20 & \boxed{-58} \end{array}$$

64. $Q(x) = x^4 - 2x^3 + 4x - 2;\ Q(-2)$ = 22

$$\begin{array}{r|rrrrr} -2 & 1 & -2 & 0 & 4 & -2 \\ & & -2 & 8 & -16 & 24 \\ \hline & 1 & -4 & 8 & -12 & 22 \end{array}$$

68. $P(z) = 2z^4 + z^2 - 3;\ P(-4)$ = 525

$$\begin{array}{r|rrrrr} -4 & 2 & 0 & 1 & 0 & -3 \\ & & -8 & 32 & -132 & 528 \\ \hline & 2 & -8 & 33 & -132 & 525 \end{array}$$

$$\begin{array}{r} 4 \\ \hline 528 \end{array}$$

Homework:

5.4 #19-25 odd; 27-43 odd; 55-61 odd