

Simplify:

$$\frac{2x(x-1)}{(x-5)(x+3)} - \frac{2}{x+3} + \frac{x}{5-x} + \frac{x}{-(x-5)}$$

$$= \frac{2x(x-1)}{(x-5)(x+3)} - \frac{2}{(x+3)} \cdot \frac{(x-5)}{(x-5)} - \frac{x}{(x-5)} \cdot \frac{(x+3)}{(x+3)}$$

$$= \frac{2x(x-1) - 2(x-5) - x(x+3)}{(x-5)(x+3)}$$

$$= \frac{2x^2 - 2x - 2x + 10 - x^2 - 3x}{(x-5)(x+3)}$$

$$= \frac{x^2 - 7x + 10}{(x-5)(x+3)} = \frac{(x-5)(x-2)}{(x-5)(x+3)} = \frac{x-2}{x+3}, x \neq -3, 5$$

### 6.3 Complex Fractions

$$6. \frac{\left(1 + \frac{1}{x}\right)}{\left(1 - \frac{1}{x^2}\right)} = \frac{\frac{x}{x} + \frac{1}{x}}{\frac{x^2}{x^2} - \frac{1}{x^2}} = \frac{\left(\frac{x+1}{x}\right)}{\left(\frac{x^2-1}{x^2}\right)}$$

$$= \frac{\cancel{x+1}}{\cancel{x}} \cdot \frac{\cancel{x^2}}{\cancel{x^2-1}} = \frac{x}{x-1}, x \neq 1, -1, 0$$

$$\begin{aligned}
 16. \quad \frac{\frac{x}{x+1} - \frac{1}{x}}{\frac{x}{x+1} + \frac{1}{x}} &= \frac{\left(\frac{x}{x+1}\right) \cdot \frac{x}{x} - \frac{1}{x} \cdot \frac{(x+1)}{(x+1)}}{\left(\frac{x}{x+1}\right) \cdot \frac{x}{x} + \frac{1}{x} \cdot \frac{(x+1)}{(x+1)}} \\
 &= \frac{\left(\frac{x^2 - x - 1}{x(x+1)}\right)}{\left(\frac{x^2 + x + 1}{x(x+1)}\right)} = \frac{x^2 - x - 1}{x(x+1)} \cdot \frac{\cancel{x(x+1)}}{x^2 + x + 1} \\
 &= \frac{x^2 - x - 1}{x^2 + x + 1}, \quad x \neq 0, -1
 \end{aligned}$$

$$\begin{aligned}
 22. \quad \frac{1 - \frac{3}{x} - \frac{10}{x^2}}{1 + \frac{11}{x} + \frac{18}{x^2}} &= \frac{\frac{x^2}{x^2} - \frac{3x}{x^2} - \frac{10}{x^2}}{\frac{x^2}{x^2} + \frac{11x}{x^2} + \frac{18}{x^2}} \\
 &= \frac{x^2 - 3x - 10}{x^2} = \frac{(x-5)(\cancel{x+2})}{\cancel{x^2} \cdot \frac{x^2}{x^2 + 11x + 18}} \\
 &= \frac{x^2 - 3x - 10}{x^2 + 11x + 18} = \frac{(x-5)(\cancel{x+2})}{(x+9)(\cancel{x+2})} \\
 &= \frac{x-5}{x+9}, \quad x \neq -9, -2, 0
 \end{aligned}$$

$$\begin{aligned}
 34. \quad & \frac{\frac{y}{y+2} - \frac{y}{y-2}}{\frac{y}{y+2} + \frac{y}{y-2}} = \frac{y(y-2) - y(y+2)}{(y+2)(y-2)} \\
 & = \frac{y^2 - 2y - y^2 - 2y}{(y+2)(y-2)} = \frac{-4y}{(y+2)(y-2)} \\
 & = \frac{-4y}{y^2 - 2y - y^2 - 2y} \cdot \frac{(y+2)(y-2)}{y^2 - 2y - y^2 - 2y} = \frac{-4y}{-4y} \cdot \frac{(y+2)(y-2)}{2y^2} \\
 & = \frac{-4y}{-4y} \cdot \frac{(y+2)(y-2)}{2y^2} = \frac{-2}{y}, \quad y \neq 0, 2, -2
 \end{aligned}$$

$$\begin{aligned}
 40. \quad & 1 - \frac{1}{\left(1 - \frac{1}{b-2}\right)} \\
 & = 1 - \frac{1}{\frac{b-2}{b-2} - \frac{1}{b-2}} = 1 - \frac{1}{\left(\frac{b-3}{b-2}\right)} \\
 & = (1) - \left(1 \cdot \frac{b-2}{b-3}\right) = \frac{b-3}{b-3} - \frac{(b-2)}{b-3} \\
 & = \frac{b-3-b+2}{b-3} = \frac{-1}{b-3}, \quad b \neq 2, 3
 \end{aligned}$$

$$\begin{aligned}
 44. \quad a - \frac{1}{2 - \frac{2}{\left(\frac{2a-2}{a}\right)}} &= a - \frac{1}{2 - \left(\frac{\cancel{2}^1}{\frac{2a-2}{a}}\right)} \\
 &= a - \frac{1}{2 - \frac{\cancel{2}^1 \cdot a}{\cancel{2}^1 \cdot \frac{2a-2}{a}}} = a - \frac{1}{\frac{a-1}{\cancel{2}^1} - \frac{a}{a-1}} \\
 &= a - \frac{1}{\left(\frac{2a-2}{a-1} - \frac{a}{a-1}\right)} = a - \frac{1}{\left(\frac{a-2}{a-1}\right)} = a - \frac{a-1}{a-2} \\
 &= \frac{a(a-2)}{a-2} - \frac{a-1}{a-2} = \frac{a^2 - 2a - a + 1}{a-2} \\
 &= \frac{a^2 - 3a + 1}{a-2}, a \neq 2, 1, 0
 \end{aligned}$$

## 6.4 Rational Equations

$$14. \quad \frac{5}{x} = \frac{2}{x+3} \quad \cancel{x(x+3)} \cdot \frac{5}{\cancel{x}} = \frac{2}{\cancel{x+3}} \cdot \cancel{x(x+3)}$$

$$5(x+3) = 2x$$

$$5x + 15 = 2x$$

$$3x = -15$$

$$x = -5$$

$$32. \left( \frac{9}{\cancel{x^2+7x+10}} \right) = \left( \frac{5}{x+2} - \frac{3}{x+5} \right) \quad (x+2)(x+5)$$

(x+2)(x+5)

$$\frac{\cancel{(x+2)(x+5)}}{1} \cdot \frac{9}{\cancel{(x+2)(x+5)}} = \frac{5}{x+2} \cdot \frac{\cancel{(x+2)(x+5)}}{1} - \frac{3}{x+5} \cdot \frac{\cancel{(x+2)(x+5)}}{1}$$

$$9 = 5(x+5) - 3(x+2)$$

$$9 = 5x + 25 - 3x - 6$$

$$9 - 25 + 6 = 2x$$

$$-10 = 2x$$

$$\cancel{-5 = x}$$

no solution

### Simplifying Rational Expressions

- find least common denominator in order to add/subtract  
[multiply by 1, i.e.  $\frac{x-1}{x-1}$ , etc.]
- dividing by a fraction = multiplying by its reciprocal
- list values excluded from domain

### Solving rational equations

- determine least common denominator & multiply both sides by LCD in order to eliminate fractions
- remember to check solutions to see if they make the original problem undefined

6.3 # 17, 23, 25, 33,  
41, 43

6.4 # 19, 25, 29, 31

\* old test # 4