Simplify:

$$\frac{x^2-12}{x^4-16}+\frac{1}{x^2-4}-\frac{1}{x^2+4}$$

$(x^2-4)(x^2+4)$ $\quad \frac{x^2-12}{(\ )(\ )}+\frac{x^2+4}{(\ )}-\frac{x^2-4}{(\ )}$

$$=\frac{x^2-12+(x^2+4)-(x^2-4)}{(x^2+4)(x^2-4)}$$

$$=\frac{x^2-4}{(x^2+4)(x^2-4)}=\frac{1}{x^2+4},\ x\neq\pm 2$$

$(x-2)(x+2)$

6.3 Complex Fractions

$\frac{1}{2}+\frac{3}{2}=\frac{4}{2}$

$$6.\ \frac{1+\frac{1}{x}}{1-\frac{1}{x^2}}=\frac{\frac{x}{x}+\frac{1}{x}}{\frac{x^2}{x^2}-\frac{1}{x^2}}=\frac{\frac{x+1}{x}}{\frac{x^2-1}{x^2}}$$

$$=\frac{x+1}{x}\cdot\frac{x^2}{x^2-1}=\frac{x}{x-1},\ x\neq 0,1,-1$$

$(x-1)(x+1)$

$\frac{x^2}{x^1}=x^{2-1}=\frac{x}{1}$

16. $\dfrac{\frac{x}{x}\cdot\frac{x}{x+1}-\frac{1}{x}\cdot\frac{x+1}{x+1}}{\frac{x}{x}\cdot\frac{x}{x+1}+\frac{1}{x}\cdot\frac{x+1}{x+1}}$

$= \dfrac{\left(\frac{x^2-(x+1)}{x(x+1)}\right)}{\left(\frac{x^2+x+1}{x(x+1)}\right)}$

$= \left(\dfrac{x^2-x-1}{x(x+1)}\right)\cdot\left(\dfrac{x(x+1)}{x^2+x+1}\right)$

$= \dfrac{x^2-x-1}{x^2+x+1}, \quad x \neq -1, 0$

22. $\dfrac{\frac{x^2}{x^2}\cdot 1-\frac{3}{x}\cdot\frac{x}{x}-\frac{10}{x^2}}{\frac{x^2}{x^2}\cdot\frac{1}{1}+\frac{11}{x}\cdot\frac{x}{x}+\frac{18}{x^2}} = \dfrac{\frac{x^2-3x-10}{x^2}}{\frac{x^2+11x+18}{x^2}}$

$= \dfrac{x^2-3x-10}{x^2}\cdot\dfrac{x^2}{x^2+11x+18}$

$= \dfrac{(x-5)(x+2)}{(x+9)(x+2)}$

$= \dfrac{x-5}{x+9}, \quad x \neq 0, -9, -2$

34. $\dfrac{\frac{y}{y+2}-\frac{y}{y-2}}{\frac{y}{y+2}+\frac{y}{y-2}}$

$\frac{y-2}{y-2}$, $\frac{y+2}{y+2}$

$= \dfrac{\frac{y(y-2)-y(y+2)}{(y-2)(y+2)}}{\frac{y(y-2)+y(y+2)}{(y-2)(y+2)}} = \dfrac{\frac{y^2-2y-y^2-2y}{(y-2)(y+2)}}{\frac{y^2-2y+y^2+2y}{(y-2)(y+2)}}$

$= \dfrac{-4y}{(y-2)(y+2)} \cdot \dfrac{(y-2)(y+2)}{2y^2} = \boxed{-\frac{2}{y},\ y \neq 0, 2, -2}$

40. $1-\dfrac{1}{\left(1\cdot\frac{b-2}{b-2}-\frac{1}{b-2}\right)} = 1-\dfrac{1}{\left(\frac{b-2-1}{b-2}\right)}$

$= 1\cdot\frac{b-3}{b-3}-1\cdot\dfrac{b-2}{(b-3)} = \dfrac{b-3-(b-2)}{b-3}$

$\frac{b-3}{b-3}-\frac{b-2}{b-3}$

$= \boxed{-\dfrac{1}{b-3},\ b \neq 3, 2}$

44. $a - \dfrac{1}{2 - \dfrac{2}{\dfrac{2a-2}{a}}} = a - \dfrac{1}{2 - \dfrac{2}{1}\cdot\dfrac{a}{2a-2}} = a - \dfrac{1}{2 - \dfrac{a}{a-1}}$

$= a - \dfrac{1}{\dfrac{2(a-1) - a}{a-1}} = a - 1\cdot\dfrac{a-1}{2a-2-a}$

$= \dfrac{a}{1}\cdot\dfrac{a-2}{a-2}\cdot\dfrac{a-1}{a-2} = \dfrac{a(a-2)-(a-1)}{a-2}$

$= \dfrac{a^2-2a-a+1}{a-2} = \dfrac{a^2-3a+1}{a-2}, a \neq 2, 1, 0$

6.4 Rational Equations

14. $\dfrac{5}{x} = \dfrac{2}{x+3}$ $\quad \dfrac{x(x+3)}{1}\cdot\dfrac{5}{x} = \dfrac{2}{x+3}\cdot\dfrac{x(x+3)}{1}$

$5(x+3) = 2x$

$5x + 15 = 2x$

$3x = -15$

$x = \dfrac{-15}{3} = -5$

32. $\frac{(x+2)(x+5)}{1}\left(\frac{9}{x^2+7x+10}\right)=\left(\frac{5}{x+2}-\frac{3}{x+5}\right)\frac{(x+2)(x+5)}{1}$

$x^2+7x+10=(x+2)(x+5)$

$\frac{9(x+2)(x+5)}{(x+2)(x+5)}=\frac{5(x+2)(x+5)}{x+2}-\frac{3(x+2)(x+5)}{x+5}$

$9=5(x+5)-3(x+2)$

$9=5x+25-3x-6$

$9-25+6=5x-3x$

$\frac{-10}{2}=\frac{2x}{2}$

$-5=x$

no solution

Simplifying Rational Expressions

- find least common denominator in order to add/subtract [multiply by 1, i.e. $\cdot\frac{x-1}{x-1}$, etc.]
- dividing by a fraction = multiplying by its reciprocal
- list values excluded from domain

Solving rational equations

- determine least common denominator & multiply both sides by LCD in order to eliminate fractions
- remember to check solutions to see if they make the original problem undefined

6.3 # 17, 23, 25, 33, 41, 43

6.4 # 19, 25, 29, 31

* old test #4