4.1 Solving Systems of Linear Equations
by Graphing and by the Substitution Method

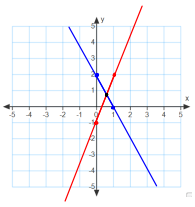
A system of equations is two or more equations considered together.

$$\begin{cases} Ax + By = C \\ Dx + Ey = F \end{cases}$$

A solution of a system of equations in two variables is an ordered pair that is a solution of each equation in the system.

A solution of a system of linear equations can be found by graphing the lines of the system on the same coordinate axes. The point of intersection of the lines is the solution of the system of equations.

$$\begin{cases} y = 3x - 1 \\ y = -2x + 2 \end{cases}$$



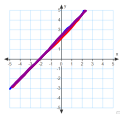
When the graphs intersect at only one point, the system of equations is called an **independent system of equations**.

(x,y)

When the lines are parallel (and do not intersect), the system of equations is called an **inconsistent system of equations**, and has no solution.

no solution

When the two equations represent the same line, the system is called a **dependent system of equations**, and has infinitely many solutions of the form (x, mx+b).



(x, mx+b) $y=mx+b$

Example 1

$$\begin{cases} y = 2x - 4 \\ y = -3x + 2 \end{cases}$$

Transitive Property of Equality
If a=b and b=c, then a=c.

$2x-4=-3x+2$

$5x=6$

$x=\frac{6}{5}$

$y=2\left(\frac{6}{5}\right)-4$

$=\frac{12}{5}-\frac{20}{5}=\frac{-8}{5}$

$\left(\frac{6}{5}, \frac{-8}{5}\right)$

Example 2

$$\begin{cases} 2x + y = 4 \\ 3x - 2y = 5 \end{cases}$$

$\Rightarrow y=-2x+4$

$3x-2(-2x+4)=5$

$3x+4x-8=5$

$7x=13$

$x=\frac{13}{7}$

$y=-2\left(\frac{13}{7}\right)+4$

$=\frac{-26}{7}+\frac{28}{7}=\frac{2}{7}$

$\left(\frac{13}{7}, \frac{2}{7}\right)$

4.1

14. $\begin{cases} 2x + 3y = 6 \\ y = -\frac{2}{3}x + 1 \end{cases}$

this is an inconsistent system of equations

parallel lines

$2x + 3\left(-\frac{2}{3}x + 1\right) = 6$

$2x - 2x + 3 = 6$

$3 = 6$, a contradiction

$\Rightarrow$ no solution

16. $\begin{cases} 3x - 2y = 6 \\ y = \frac{3}{2}x - 3 \end{cases}$

these equations represent the same line

dependent system

$3x - 2\left(\frac{3}{2}x - 3\right) = 6$

$3x - 3x + 6 = 6$

$6 = 6$, an identity

$\Rightarrow$ infinitely many solution

$\left(x, \frac{3}{2}x - 3\right)$

4.2 Solving Systems of Equations by the Elimination Method

(Addition Method)

Rules: We can...

1. multiply an equation by a non-zero constant
2. interchange any 2 equations
3. add a non-zero multiple of any equation to another

16. $\begin{cases} 3x + 4y = 25 \\ 2x + y = 10 \end{cases}$ (-4)

Multiplying 2nd equation by -4 yields the equivalent system:

$$\begin{array}{r} 3x + 4y = 25 \\ +(-8x - 4y = -40) \\ \hline -5x \quad = -15 \end{array}$$

$x = 3$

$2(3) + y = 10$

$6 + y = 10$

$y = 4$

$(3, 4)$

14. $\begin{cases} 3x + 6y = 7 \\ 2x + 4y = 5 \end{cases}$ (2) (-3)

$\Rightarrow 6x + 12y = 14$

$+ \quad -6x - 12y = -15$

$0 = -1$ ↯ contradiction

no solution

26. $\begin{cases} 3x + 3y = y + 1 \\ x + 3y = 9 - x \end{cases}$

$\Rightarrow (3x + 2y = 1)(2) \Rightarrow 6x + 4y = 2$

$(2x + 3y = 9)(-3) + -6x - 9y = -27$

$-5y = -25$

$y = 5$

$3x + 2(5) = 1$

$3x + 10 = 1$

$3x = -9$

$x = -3$

$(-3, 5)$