5.1 Exponential Expressions

$x^n = \underbrace{x \cdot x \cdot x \cdots x}_{n \text{ times}}$

For $m, n, p \in \mathbb{Z}$ (integers),

$x^m x^n = x^{m+n}$

$x^2 x^3 = x^5$

$\frac{x^m}{x^n} = x^{m-n}$

$\frac{x^2}{x^3} = x^{2-3} = x^{-1}$

$(x^m)^n = x^{mn}$

$(x^2)^3 = x^6$

$(x^m y^n)^p = x^{mp} y^{np}$

$\left(\frac{x^m}{y^n}\right)^p = \frac{x^{mp}}{y^{np}}$, $y \neq 0$

$x^0 = 1$, $x \neq 0$

$x^{-n} = \frac{1}{x^n}$, $x \neq 0$

$\frac{1}{x^{-n}} = x^n$

A simplified exponential expression contains:

- only one instance of each variable
- no negative exponents

4. $(-2ab^4)(-3a^2b^4) = (-2)(-3)a \cdot a^2 \cdot b^4 b^4$

$= \boxed{6a^3b^8}$

20. $[(3x^2y^3)^2]^2 = [3^2 \cdot (x^2)^2 (y^3)^2]^2 = [9x^4y^6]^2$

$= 9^2 (x^4)^2 (y^6)^2 = \boxed{81x^8y^{12}}$

66. $\dfrac{6^2a^{-2}b^3}{3a^1b^4} = \dfrac{36}{3a^{1-(-2)}b^{4-3}} = \boxed{\dfrac{12}{a^3b}}$

$\dfrac{a^{-2}}{a^1} = \dfrac{a^{-2-1}}{1} = \dfrac{1}{a^{1-(-2)}}$

$= a^{-3} \qquad = \dfrac{1}{a^3}$

72. $\left(\dfrac{x^{-3}y^{-4}}{x^{-2}y^1}\right)^{-2} = \left(\dfrac{1}{x^{-2-(-3)}y^{1-(-4)}}\right)^{-2}$

$= \left(\dfrac{1}{x^1y^5}\right)^{-2} = \dfrac{(1)^{-2}}{(x^1)^{-2}(y^5)^{-2}} = \dfrac{1}{x^{-2}y^{-10}}$

$1^{-2} = \dfrac{1}{1^2} = \dfrac{1}{1} = 1 \qquad = \boxed{x^2y^{10}}$

80. $\left(\dfrac{4^{-2}xy^{-3}}{x^{-3}y}\right)^{3}\left(\dfrac{8^{-1}x^{-2}y}{x^{4}y^{-1}}\right)^{-2}$

$$=\left(\frac{x^{1-(-3)}}{4^{2}y^{1-(-3)}}\right)^{3}\left(\frac{y^{1-(-1)}}{8^{1}x^{4-(-2)}}\right)^{-2}=$$

$$=\left(\frac{x^{4}}{16y^{4}}\right)^{3}\left(\frac{y^{2}}{8x^{6}}\right)^{-2}=\frac{(x^{4})^{3}}{(2^{4})^{3}(y^{4})^{3}}\cdot\frac{(y^{2})^{-2}}{(2^{3})^{-2}(x^{6})^{-2}}$$

$$=\frac{x^{12}y^{-4}}{2^{12}y^{12}2^{-6}x^{-12}}=\frac{x^{12-(-12)}}{2^{12-6}y^{12-(-4)}}$$

$$=\frac{x^{24}}{2^{6}y^{16}}=\boxed{\frac{x^{24}}{64y^{16}}}$$

80. $\left(\dfrac{4^{-2}xy^{-3}}{x^{-3}y}\right)^{3}\left(\dfrac{8^{-1}x^{-2}y}{x^{4}y^{-1}}\right)^{-2}$

$$=\left(\frac{4^{-6}x^{3}y^{-9}}{x^{-9}y^{3}}\right)\left(\frac{8^{2}x^{4}y^{-2}}{x^{-8}y^{2}}\right)$$

$$=\frac{x^{3-(-9)}}{4^{6}y^{3-(-9)}}\cdot\frac{8^{2}x^{4-(-8)}}{y^{2-(-2)}}$$

$$=\frac{x^{12}}{4^{6}y^{12}}\cdot\frac{8^{2}x^{12}}{y^{4}}=\frac{8^{2}}{4^{6}}\frac{x^{24}}{y^{16}}$$

$$\frac{(2^{3})^{2}}{(2^{2})^{6}}=\frac{2^{6}}{2^{12}}=\frac{1}{2^{6}}$$

$$=\boxed{\frac{x^{24}}{64y^{16}}}$$

5.2 Introduction to Polynomials

A polynomial is an expression consisting of variables and constants, using only the operations of addition, subtraction, multiplication, and non-negative integer exponents.

A polynomial with one term is a monomial.

e.g. $3x^2$ or $5xy^3$

A polynomial with two terms is a binomial.

e.g. $7xy^5 - 3x$ or $xyzw + 23x^2$ or $x - 2$

A polynomial with three terms is a trinomial.

e.g. $x^2 + 5x - 6$

The degree of a monomial is the sum of the exponents of the variables.

$7xy^5$ has degree 6

$xyzw$ has degree 4

$13x^3yz^2$ has degree 6

$-2ab^3$ has degree 4

The degree of a polynomial is the greatest of the degrees of any of its terms.

$x^2 + 5x - 6$ has degree 2

$\underbrace{3xy}_{2} - \underbrace{15x^3y}_{4} + \underbrace{2v^3xz}_{5}$ has degree 5

$\underbrace{15xy^2}_{3} - \underbrace{\sqrt{2}x}_{1} + \underbrace{32xyz}_{3} - \underbrace{5000}_{0}$ has degree 3

The terms of a polynomial in only one variable are usually arranged in descending order, so that the exponents of the variable decrease from left to right, in the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_2 x^2 + a_1 x^1 + a_0 x^0$$

$x^0 = 1$

$a_n, \ldots, a_0$ are real-numbered coefficients

$a_n x^n$ is the lead term (term containing the variable with the largest exponent)

a_n is the leading coefficient (coefficient of the variable with the largest exponent)

a_0 is the constant term (term without a variable)

n is the degree of the polynomial (largest exponent)

constant functions $f(x) = c$

The linear function $f(x) = mx + b$ is a polynomial of degree one.

A second-degree polynomial of the form $f(x) = ax^2 + bx + c$ is called a quadratic function.

A third-degree polynomial is called a cubic function.

$f(x) = ax^3 + bx^2 + cx + d$

Problems from Section 5.2:

Is it a polynomial? If so, state the lead term, leading coefficient, degree, and constant term.

16. $P(x) = 3x^4 - 3x - 7$

Lead term: $3x^4$

Leading coefficient: 3

Degree: 4

Constant term: -7

18. $R(x) = \frac{3x^2 - 2x + 1}{x}$

Lead term:

Leading coefficient:

Degree:

Constant term:

Not a polynomial

20. $f(x) = x^2 - \sqrt{x+2} + 8$

Lead term:

Leading coefficient:

Degree:

Constant term:

Not a polynomial

22. $g(x) = -4x^5 + 3x^2 + x - \sqrt{7}$

Lead term: $-4x^5$

Leading coefficient: -4

Degree: 5

Constant term: $-\sqrt{7}$

$f(x) = 2x^3 - 3x^2 - x^5 + (\sqrt{3})x$

lead term: $-x^5$

leading coeff: -1

degree: 5

Constant term: 0

To evaluate a polynomial, replace the variable by its value and simplify.

6. Given $R(x) = -x^3 + 2x^2 - 3x + 4$, evaluate $R(-1)$.

$R(-1) = -(-1)^3 + 2(-1)^2 - 3(-1) + 4 =$

$= -(-1) + 2(1) + 3 + 4 =$

$= 1 + 2 + 7 = \boxed{10}$

Polynomials can be added by combining like terms.

36. $(3x^2 - 2x + 7) + (-3x^2 + 2x - 12)$

$3x^2 + (-3x^2) + (-2x) + 2x + 7 + (-12)$

$= 0 + 0 + (-5) = \boxed{-5}$

42. $(3a^2 - 9a) - (-5a^2 + 7a - 6)$

$3a^2 - 9a + 5a^2 - 7a + 6$

$= \boxed{8a^2 - 16a + 6}$

50. $(2x^4 - 2x^2 + 1) - (3x^3 - 2x^2 + 3x + 8)$

$2x^4 - 3x^3 - 3x - 7$

46. $(2x^{2n} - x^n - 1) - (5x^{2n} + 7x^n + 1)$

$-3x^{2n} - 8x^n - 2$

Homework:

Problems from 5.1 and 5.2 on Khan Academy:

5.1	Exponential Expressions	• Positive and zero exponents • Negative exponents • Patterns in zeros • Exponent rules • Scientific notation intuition • Scientific notation • Multiplying and dividing scientific notation • Orders of magnitude • Simplifying expressions with exponents
5.2	Introduction to Polynomials	• Adding and subtracting polynomials