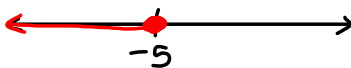Review Problems:

1. Write in interval notation: $\{x|x \le -5\}$

$(-\infty, -5]$

-5

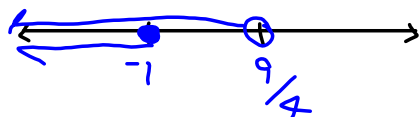2. Solve. Write the solution set in interval notation.

$6 - 4x > -3$ and $3x + 2 \le -1$

$-4x > -9$ $3x \le -3$

$x < \frac{9}{4}$ $x \le -1$

$(-\infty, -1]$

-1   9/4

3. Simplify. $(x^5 x^{-3})^{-2}(x^{-4}x^3)^4$

$(x^m)^n = x^{mn}$

$x^{-10} x^6 \quad x^{-16} \quad x^{12}$

$= x^{-4} x^{-4}$

$x^m x^n = x^{m+n}$

$= x^{-10+6-16+12}$

$= x^{-8} = \dfrac{1}{x^8}$

Simplify.

108. $\dfrac{(2x+1)^5}{(2x+1)^3}$

$\dfrac{x^m}{x^n} = x^{m-n}$

$= (2x+1)^2 = 4x^2 + 4x + 1$

$(2x+1)(2x+1)$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)(a+b) =$

$a^2 + ab + ab + b^2$

116. $[x + (y+1)][x - (y+1)]$

$= x^2 - (y+1)^2 = x^2 - (y^2 + 2y + 1)$

$(a+b)(a-b)$

$= a^2 - b^2$

$(y+1)(y+1)$

$= x^2 - y^2 - 2y - 1$

126. Find $(3n^4)^3$ if $5(n-1) = 2(3n-2)$.

$3^3 n^{12}$

$5n - 5 = 6n - 4$

$27 n^{12}$

$-5 + 4 = 6n - 5n$

$-1 = n$

$27(-1)^{12} = 27$

Orders of magnitude

$a \times 10^n$
$1 \leq a < 10$
$n \in \mathbb{Z}$

Rewrite 4×10^4 in decimal notation.

$$40000$$

Multiplying and dividing scientific notation

Express this quotient in scientific notation: $\dfrac{5.900 \times 10^{11}}{1.0 \times 10^5}$

$$5.9 \times 10^6$$

How many times smaller is 1×10^{-9} than 7×10^{-6}?

.000000001   .000007

$$\frac{7 \times 10^{-6}}{1 \times 10^{-9}} = 7 \times 10^{-6-(-9)} = 7 \times 10^3 = 7000$$

The length of a water molecule is 0.0000000000278 meters. Estimate the length of a water molecule by rewriting it in the form of $x \times 10^y$ meters, where x and y are whole numbers.

$$2.78 \times 10^{-10} \approx 3 \times 10^{-10}$$

5.4 Division of Polynomials

Long Division $57 \div 2$

$$\begin{array}{r} 28 \\ 2\overline{)57} \\ -4 \\ \hline 17 \\ -16 \\ \hline 1 \end{array}$$

$\Rightarrow 57 = 2(28) + 1$

$$\frac{57}{2} = 28 + \frac{1}{2}$$

4. $(x^2 - 14x + 24) \div (x - 2) = \boxed{x - 12}$

$$
\begin{array}{r}
x - 12 \\
x - 2 \overline{\smash{\big)}\ x^2 - 14x + 24} \\
-(x^2 - 2x) \\
\hline
-12x + 24 \\
-(-12x + 24) \\
\hline
0
\end{array}
$$

$\dfrac{x^2}{x} \approx x$

$\Rightarrow$
$x^2 - 14x + 24 =$
$(x-2)(x-12)$

6. $(x^3 + 4x^2 - 8) \div (x + 4) = \boxed{x^2 + \dfrac{-8}{x+4}}$

$$
\begin{array}{r}
x^2 \\
x + 4 \overline{\smash{\big)}\ x^3 + 4x^2 - 8} \\
-(x^3 + 4x^2) \\
\hline
-8
\end{array}
$$

$\dfrac{x^3}{x} \approx x^2$

$x^3 + 4x^2 - 8 =$
$(x+4)(x^2) - 8$

10. $(18x^2 - 3x + 2) \div (3x + 2) = \boxed{6x - 5 + \dfrac{12}{3x+2}}$

$$
\begin{array}{r}
6x - 5 \\
3x + 2 \overline{\smash{\big)}\ 18x^2 - 3x + 2} \\
-(18x^2 + 12x) \\
\hline
-15x + 2 \\
-(-15x - 10) \\
\hline
12
\end{array}
$$

Quotient: $6x - 5$

Remainder: 12

14. $(12x^4 - 11x^2 + 10) \div (3x^2 + 1) = \boxed{4x^2 - 5 + \dfrac{15}{3x^2+1}}$

$$
\begin{array}{r}
4x^2 - 5 \\
3x^2 + 1 \overline{\smash{\big)}\ 12x^4 - 11x^2 + 10} \\
-(12x^4 + 4x^2) \\
\hline
-15x^2 + 10 \\
-(-15x^2 - 5) \\
\hline
15
\end{array}
$$

$5 \div 2 = 2 + \dfrac{1}{2}$

Quotient:
$4x^2 - 5$

Remainder:
15

20. $\dfrac{x + 3x^4 - x^2 + 5x^3 - 2}{x + 2}$

$$x+2 \overline{\smash{)}3x^4 + 5x^3 - x^2 + x - 2}$$

24. $\dfrac{2 - 3x^2 + 5x^3}{x^2 + 3}$

$$x^2+3 \overline{\smash{)}5x^3 - 3x^2 + 2}$$

Synthec Division

only works when dividing by $(x \pm a)$

28. $(3x^2 + 19x + 20) \div (x + 5)$ = $\boxed{3x+4}$

$$\begin{array}{r|rrr} -5 & 3 & 19 & 20 \\ & & -15 & -20 \\ \hline & 3 & 4 & \boxed{0} \end{array}$$

linear coeff.   constant   remainder

$\dfrac{3x^2}{x} \approx 3x$

30. $(4x^2 - 8) \div (x - 2)$ = $\boxed{4x+8 + \dfrac{8}{x-2}}$

missing x-term

$$\begin{array}{r|rrr} 2 & 4 & 0 & -8 \\ & & 8 & 16 \\ \hline & 4x & 8 & \boxed{8} \end{array}$$

const.   remainder

Quotient:
$4x + 8$

Remainder:
8

$\dfrac{4x^2}{x} \approx 4x$

4

34. $(3x^2 - 15) \div (x + 3) = \boxed{3x - 9 + \dfrac{12}{x+3}}$

$$\begin{array}{r|rrr} -3 & 3 & 0 & -15 \\ & & -9 & 27 \\ \hline & 3 & -9 & \boxed{12} \end{array}$$

38. $(x^3 - 4x^2 + x + 6) \div (x + 1) = \boxed{x^2 - 5x + 6}$

$$\begin{array}{r|rrrr} -1 & 1 & -4 & 1 & 6 \\ & & -1 & 5 & -6 \\ \hline & 1 & -5 & 6 & \boxed{0} \end{array}$$

42. $(x^3 + 2x + 5) \div (x - 2)$

48. $\dfrac{3 - 13x - 5x^2 + 9x^3 - 2x^4}{3 - x}$

52. $\dfrac{x^4 - 3x^3 - 30}{x + 2}$

Evaluate a Polynomial using Synthetic Division

Remainder Theorem: If the polynomial $P(x)$ is divided by $x - a$, the remainder is $P(a)$.

56. $Q(x) = 3x^2 - 5x - 1; \ Q(2)$

60. $R(t) = 3t^3 + t^2 - 4t + 2; \ R(-3)$

64. $Q(x) = x^4 - 2x^3 + 4x - 2;\ \ Q(-2)$

68. $P(z) = 2z^4 + z^2 - 3;\ \ P(-4)$

Textbook Homework Problems:

5.4 #19-25 odd; 27-43 odd; 55-61 odd