

6.1, 6.2 Rational Functions

A rational function is of the form $f(x) = \frac{p(x)}{q(x)}$, where p & q are polynomials.

Simplify. Factor First!

$$46. \quad \frac{3a^2 - 6a}{12 - 6a} = \frac{\cancel{3a} (\cancel{a-2})}{\cancel{-6} (\cancel{-2+a})}$$

$$= \frac{a}{-2} = \frac{-a}{2} = \boxed{\frac{-a}{2}},$$

$a \neq 2$

$$\frac{a^1}{a^1} = a^{1-1} = a^0 = 1$$

$$\begin{aligned}
 52. \quad & \frac{3x^3y^3 - 12x^2y^2 + 15xy}{3xy} \\
 & = \frac{\cancel{3xy}(x^2y^2 - 4xy + 5)}{\cancel{3xy}} \\
 & = \boxed{x^2y^2 - 4xy + 5}, \quad x \neq 0, y \neq 0
 \end{aligned}$$

$$\begin{aligned}
 60. \quad & \frac{\begin{array}{l} 2x(x+4y) - y(x+4y) \\ 2x^2 + 8xy - xy - 4y^2 \end{array}}{4x^2 - 4xy + y^2} = \frac{(x+4y)\cancel{(2x-y)}}{(2x-y)\cancel{(2x-y)}} \\
 & \frac{\begin{array}{l} 4x^2 - 2xy - 2xy + y^2 \\ 2x(2x-y) - y(2x-y) \end{array}}{4x^2 - 4xy + y^2} = \frac{x+4y}{2x-y}, \quad y \neq 2x \\
 & \begin{array}{l} 2x-y \neq 0 \\ 2x \neq y \\ x \neq \frac{y}{2} \end{array}
 \end{aligned}$$

$$\begin{aligned}
 72. \quad & \frac{4a^2 - 8ab + 4b^2}{4a^2 - 4b^2} \\
 &= \frac{4(a^2 - 2ab + b^2)}{4(a^2 - b^2)} \\
 &= \frac{\cancel{4}(a-b)(\cancel{a-b})}{\cancel{4}(a+b)(\cancel{a-b})} = \frac{a-b}{a+b}, a \neq \pm b \\
 & \quad a+b \neq 0, a-b \neq 0 \\
 & \quad a \neq -b, a \neq b
 \end{aligned}$$

$$\begin{aligned}
 78. \quad & \frac{x^4 - 2x^2 - 3}{x^4 + 2x^2 + 1} = \frac{(x^2 - 3)(\cancel{x^2 + 1})}{(x^2 + 1)(\cancel{x^2 + 1})} \\
 &= \frac{x^2 - 3}{x^2 + 1}
 \end{aligned}$$

6.2 Operations on Rational Functions

(Same rules of fractions apply)

$$\frac{a}{b} \pm \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{d} \pm \frac{c}{d} \cdot \frac{b}{b} = \frac{ad \pm cb}{bd}$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

$$\begin{aligned}
 & \begin{array}{l} 2x(x-1)-3(x-1) \\ 2x^2-2x-3x+3 \end{array} \\
 8. & \frac{2x^2-5x+3}{x^6y^3} \cdot \frac{x^4y^4}{2x^2-x-3} \\
 & \begin{array}{l} 2x^2+2x-3x-3 \\ 2x(x+1)-3(x+1) \end{array} \\
 & = \frac{(x-1)(\cancel{2x-3})}{\cancel{x^2}y^3} \cdot \frac{\cancel{x^1}y^{\cancel{4}}}{(x+1)(\cancel{2x-3})} \\
 & = \boxed{\frac{y(x-1)}{x^2(x+1)}} = \frac{xy-y}{x^3+x^2}
 \end{aligned}$$

$$\frac{x^4}{x^6} = \frac{1}{x^{6-4}}$$

$$\begin{aligned}
 2x-3 & \neq 0 \\
 2x & \neq 3 \\
 x & \neq \frac{3}{2}
 \end{aligned}$$

$$x \neq 0, -1, \frac{3}{2}$$

$$18. \frac{\begin{matrix} (x-2)(x+2)(x-1)(x+1) \\ (x^2-4)(x^2-1) \end{matrix}}{3x^2-4x-4} \cdot \frac{\begin{matrix} 3x(x-4)+2(x-4) \\ 3x^2-12x+2x-8 \end{matrix}}{x^2-4}$$

$$= \frac{\begin{matrix} 3x^2-6x+2x-4 \\ 3x(x-2)+2(x-2) \end{matrix}}{3x^2-4x-4} \cdot \frac{\begin{matrix} 3x^2-10x-8 \\ (x-2)(x+2) \end{matrix}}{x^2-4}$$

$$= \frac{\cancel{(x-2)}\cancel{(x+2)}\cancel{(x-1)}\cancel{(x+1)}(x-4)\cancel{(3x+2)}}{\cancel{(x-2)}\cancel{(3x+2)}\cancel{(x-2)}\cancel{(x+2)}}$$

$$= \frac{(x-1)(x+1)(x-4)}{x-2}, \quad x \neq 2, -2, -\frac{2}{3}$$

$$34. \frac{16x^2-9}{6-5x-4x^2} \div \frac{16x^2+24x+9}{4x^2+11x+6}$$

$$= \frac{\begin{matrix} (4x)^2-3^2 \\ -1(4x^2+5x-6) \\ -1(4x^2+8x-3x-6) \end{matrix}}{6-5x-4x^2} \div \frac{16x^2+12x+12x+9}{4x^2+8x+3x+6}$$

$$= \frac{\begin{matrix} (4x-3)(4x+3) \\ -1[4x(x+2)-3(x+2)] \end{matrix}}{6-5x-4x^2} \cdot \frac{4x(4x+3)+3(4x+3)}{4x(x+2)+3(x+2)}$$

$$= \frac{(4x-3)(4x+3)}{-1(x+2)(4x-3)} \div \frac{(4x+3)(4x+3)}{(x+2)(4x+3)}$$

$$= \frac{\cancel{(4x-3)}\cancel{(4x+3)}}{-1\cancel{(x+2)}\cancel{(4x-3)}} \cdot \frac{\cancel{(x+2)}\cancel{(4x+3)}}{\cancel{(4x+3)}\cancel{(4x+3)}} = -1$$

$$= -1, \quad x \neq -2, \frac{3}{4}, -\frac{3}{4}$$

$$\begin{aligned}
36. \quad & \frac{x^{4n}-1}{x^{2n}+x^n-2} \div \frac{x^{2n}+1}{x^{2n}+3x^n+2} \\
& = \frac{(x^{2n}-1)(x^{2n}+1)}{(x^n+2)(x^n-1)} \div \frac{x^{2n}+1}{(x^n+2)(x^n+1)} \\
& = \frac{\cancel{(x^n-1)}(x^n+1)\cancel{(x^{2n}+1)}}{\cancel{(x^n+2)}\cancel{(x^n-1)}} \cdot \frac{\cancel{(x^n+2)}(x^n+1)}{\cancel{x^{2n}+1}} \\
& = \boxed{(x^n+1)(x^n+1)} = (x^n+1)^2
\end{aligned}$$

Textbook Problems:

6.1 #39-79 odd

6.2 #3-95 odd

Khan Academy:

- Dividing polynomials by binomials 1
- Simplifying rational expressions 1
- Dividing polynomials by binomials 2
- Simplifying rational expressions 2
- Dividing polynomials by binomials 3
- Simplifying rational expressions 3
- Adding and subtracting rational expressions 0.5
- Adding and subtracting rational expressions 1
- Adding and subtracting rational expressions 1.5
- Adding and subtracting rational expressions 2
- Multiplying and dividing rational expressions 1
- Multiplying and dividing rational expressions 2