7.2 Operations on Radical Expressions

Properties of Radicals:

Let $m, n \in \mathbb{N}$ and $a, b \in \mathbb{R}$ such that $a^{\frac{1}{n}}, b^{\frac{1}{n}} \in \mathbb{R}$

$a^{1/n} b^{1/n} = (ab)^{1/n}$

$\sqrt[n]{b} = b^{\frac{1}{n}}$

$\left(\sqrt[n]{b}\right)^m = \sqrt[n]{b^m} = b^{\frac{m}{n}}$

$\sqrt[n]{b^n} = \begin{cases} b, & if\ n\ is\ odd \\ |b|, & if\ n\ is\ even \end{cases}$

$\dfrac{a^{1/n}}{b^{1/n}} = \left(\dfrac{a}{b}\right)^{1/n}$

$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

$\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$

$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

$a^{1/mn} = \left(a^{1/n}\right)^{1/m} = \sqrt[m]{\sqrt[n]{a}}$

Add & Subtract Radicals by Combining Like Terms

7.2

26. $3\sqrt[3]{11} - 8\sqrt[3]{11}$

$= \boxed{-5\sqrt[3]{11}}$

32. $\sqrt{48x} + \sqrt{147x}$

$16\cdot 3$

$\sqrt{4^2\cdot 3x} + \sqrt{7^2\cdot 3x}$

$4\sqrt{3x} + 7\sqrt{3x}$

$= \boxed{11\sqrt{3x}}$

$7\overline{)147}$ = 21

$147 = 7\cdot 7\cdot 3$

34. $\sqrt{18b} + \sqrt{75b}$

$9\cdot 2 \quad 25\cdot 3$

$\sqrt{3^2\cdot 2b} + \sqrt{5^2\cdot 3b}$

$\boxed{3\sqrt{2b} + 5\sqrt{3b}}$

$\sqrt[m]{x^n}$ look for the largest multiple of m, $\leq n$

46. $2b\sqrt[3]{16b^2} + \sqrt[3]{128b^5}$

$8\cdot 2$

$2b\sqrt[3]{2^3\cdot 2b^2} + \sqrt[3]{4^3\cdot 2\cdot b^3 b^2}$

$4b\sqrt[3]{2b^2} + 4b\sqrt[3]{2b^2}$

$= \boxed{8b\sqrt[3]{2b^2}}$

$128 = 2\cdot 64$

$2\cdot 2\cdot 32$

$2\cdot 2\cdot 16\cdot 2$

$2\cdot 2\cdot 8\cdot 2\cdot 2$

$2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2$

2^7

$2^6\cdot 2$

$(2^2)^3\cdot 2$

62. $\sqrt{a^3b}\sqrt{ab^4}$

$= \sqrt{(a^3b)(ab^4)}$

$= \sqrt{a^4 \cdot b \cdot b^4}$

$= \sqrt{(a^2)^2 \cdot b (b^2)^2}$

$= \boxed{a^2b^2\sqrt{b}}$

64. $\sqrt{5x^3y}\sqrt{\underset{5\cdot 2}{10}x^3y^4}$

$= \sqrt{5 \cdot 5 \cdot 2 \cdot x^3 x^3 \cdot y \cdot y^4}$

$= \sqrt{5^2 \cdot 2 \cdot (x^3)^2 \cdot y (y^2)^2}$

$= \boxed{5|x^3|y^2\sqrt{2y}}$

74. $\sqrt{3a}\left(\sqrt{27a^2}-\sqrt{a}\right)$; $a>0$

$= \sqrt{3a}\sqrt{27a^2} - \sqrt{3a}\sqrt{a}$

$\quad 3 \cdot 9 \cdot 3$

$= \sqrt{9^2 \cdot a \cdot a^2} - \sqrt{3a^2}$

$= \boxed{9a\sqrt{a} - a\sqrt{3}}$

82. $(\sqrt{2}-3)(\sqrt{2}+4)$

$= \sqrt{2}\cdot\sqrt{2} + 4\sqrt{2} - 3\sqrt{2} - 3(4)$

$\quad \sqrt{2^2}$

$= 2 + \sqrt{2} - 12$

$= \boxed{\sqrt{2} - 10}$

Rationalize the Denominator

(rewrite so that there are no radicals in the denominator)

100. $\frac{2}{\sqrt{3y}} \cdot \frac{\sqrt{3y}}{\sqrt{3y}} = \frac{2\sqrt{3y}}{\sqrt{(3y)^2}} = \boxed{\frac{2\sqrt{3y}}{3y}}$

102. $\frac{9}{\sqrt{3a}} \cdot \frac{\sqrt{3a}}{\sqrt{3a}} = \frac{\overset{3}{\cancel{9}}\sqrt{3a}}{\cancel{3}a} = \boxed{\frac{3\sqrt{3a}}{a}}$

assume all variables are positive

When the *entire* denominator is a radical, multiply by $1 = \frac{\sqrt{\square}}{\sqrt{\square}}$

When the denominator contains radicals added or subtracted with something, we multiply by the conjugate of the denominator over itself. $a + b$ and $a - b$ are conjugates.

114. $\frac{5}{(2-\sqrt{7})} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{5\sqrt{7}}{2\sqrt{7}-7}$

$(a+b)(a-b)$
$= a^2 - b^2$

$\frac{5}{(2-\sqrt{7})} \cdot \frac{(2+\sqrt{7})}{(2+\sqrt{7})} = \frac{10+5\sqrt{7}}{4+\cancel{2\sqrt{7}}-\cancel{2\sqrt{7}}-\sqrt{7}\sqrt{7}}$

$= \frac{10+5\sqrt{7}}{4-7} = \boxed{\frac{10+5\sqrt{7}}{-3}}$

120. $\dfrac{(3-\sqrt{x})}{(3+\sqrt{x})} \cdot \dfrac{(3-\sqrt{x})}{(3-\sqrt{x})} = \dfrac{9-3\sqrt{x}-3\sqrt{x}+x}{9-\cancel{3\sqrt{x}}+\cancel{3\sqrt{x}}-x}$

$= \boxed{\dfrac{9-6\sqrt{x}+x}{9-x}}$

122. $\dfrac{(\sqrt{2}+\sqrt{3})}{(\sqrt{3}-\sqrt{2})} \cdot \dfrac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})}$

$= \dfrac{\sqrt{2}\sqrt{3}+\sqrt{2}\sqrt{2}+\sqrt{3}\sqrt{3}+\sqrt{3}\sqrt{2}}{\sqrt{3}\sqrt{3}+\cancel{\sqrt{3}\sqrt{2}}-\cancel{\sqrt{2}\sqrt{3}}-\sqrt{2}\sqrt{2}}$

$= \dfrac{\sqrt{6}+2+3+\sqrt{6}}{3-2}$

$= \boxed{2\sqrt{6}+5}$

8.2 Solving Quadratic Equations using the Quadratic Formula

Given a quadratic equation in standard form,

$$ax^2 + bx + c = 0, \quad a \neq 0$$

$\boxed{\text{Quadratic Formula: } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$

60. $z^2 - 4z - 8 = 0$

$a = 1 \quad b = -4 \quad c = -8$

$$z = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-8)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 + 32}}{2} = \frac{4 \pm \sqrt{48}}{2}$$

$\sqrt{48}$
$= \sqrt{16 \cdot 3}$
$= \sqrt{4^2 \cdot 3}$

$$= \frac{4 \pm 4\sqrt{3}}{2} = \frac{\cancel{2}(2 \pm 2\sqrt{3})}{\cancel{2}} = \boxed{2 \pm 2\sqrt{3}}$$

$2 + 2\sqrt{3}$ & $2 - 2\sqrt{3}$

68. $4p^2 - 7p = -3$

$4p^2 - 7p + 3 = 0$

$a = 4, \; b = -7, \; c = 3$

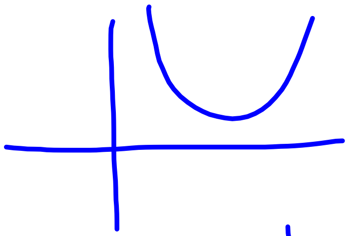
$$p = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(4)(3)}}{2(4)}$$

$$= \frac{7 \pm \sqrt{49 - 48}}{8} = \frac{7 \pm \sqrt{1}}{8} = \frac{7 \pm 1}{8}$$

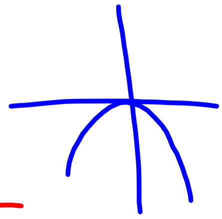
$$= \begin{cases} \frac{7+1}{8} = \frac{8}{8} = \boxed{1} \\ \frac{7-1}{8} = \frac{6}{8} = \boxed{\frac{3}{4}} \end{cases}$$

$$x^2 + x + 2 = 0$$

$a=1, \ b=1, \ c=2$

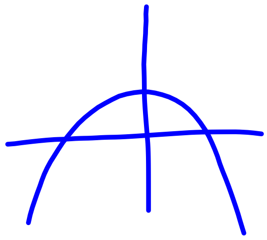


$$x = \frac{-1 \pm \sqrt{1^2-4(1)(2)}}{2(1)}$$



$$= \frac{-1 \pm \sqrt{1-8}}{2} = \frac{-1 \pm \sqrt{-7}}{2}$$

$$\sqrt{-1} = i$$



Wednesday: turn in all textbook homework from Ch 5 & 6
I'm not looking for ALL the problems, just if you completed *some* problems from *each* section.

5.1 #63-85 odd
5.2 #3-7odd, 15-25odd, 35-49odd
5.3 #25-29odd, 43-51odd, 61-67odd, 89-97odd, 109-117odd
5.4 #19-25 odd; 27-43 odd; 55-61 odd
5.5 #21-47 odd, 79-137odd
5.6 #3-131 odd
5.7 #35-49 odd; 51-57 odd; 61-75 odd
Ch 5 Review/Test/Cumulative Review
6.1 #39-79 odd
6.2 #3-95 odd
6.3 #17, 23, 25, 33, 41, 43
6.4 #19, 25, 29, 31
6.6 # 5-25 odd

7.1 #85-113 odd; 39-73 odd; 125-149 odd
7.2 #11-21 odd
New Homework:
7.2 #43-51odd; 57-65odd;85-91 odd; 97-103odd; 113-121odd
8.2 #59-69odd