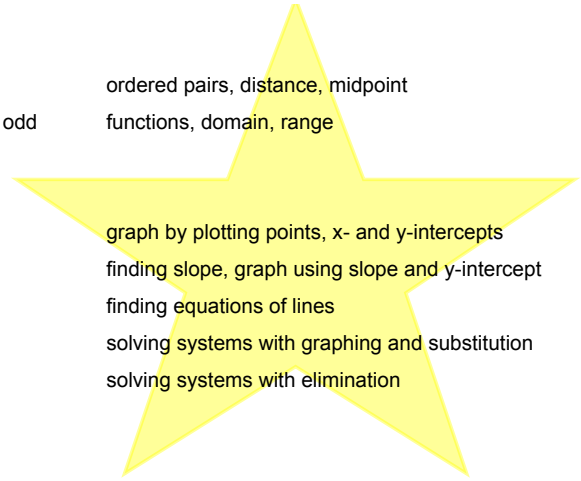


HW#3 - Due Tuesday, 09/08:

3.1 - #3-29 odd	ordered pairs, distance, midpoint
3.2 - #3-16 all, 21-43 odd, 49-87 odd	functions, domain, range

HW#4 - Due Friday, 09/11:

3.3 - #3-9 odd, 15-33 odd	graph by plotting points, x- and y-intercepts
3.4 - #3-19 odd, 29-41 odd	finding slope, graph using slope and y-intercept
3.5 - #3-49 odd	finding equations of lines
4.1 - #9,11,13,15,29,37,43,49	solving systems with graphing and substitution
4.2 - #9,13,17,25,27,31,35	solving systems with elimination

Expect a Quiz VERY SOON on:

- midpoint
- distance
- slope
- x- and y-intercepts
- equation of a line
- functions
- domain
- range

Sections 3-3-3.6 Linear Functions

Point-slope formula:

$$\boxed{y - y_1 = m(x - x_1)}$$

Note that rearranged, this looks like:

$$m = \frac{y - y_1}{x - x_1}$$

Recall:

Slope-intercept equation:

$y = mx + b$

Standard Equation:

$Ax + By = C$

Find the slope-intercept (y=mx+b) equation of the line:

1. slope 2 ; passing through (3,7)

m x_1, y_1

$y - 7 = 2(x - 3)$

$y - 7 = 2x - 6$

$\boxed{y = 2x + 1}$

in standard form

$2x - y = -1$

2. passes through (-5, 2) & (6, -1)

$x_1 y_1$ $x_2 y_2$

$m = \frac{-1-2}{6-(-5)} = \frac{-3}{11}$

$y - (-1) = \frac{-3}{11}(x - 6)$

$y + 1 = -\frac{3}{11}x + \frac{18}{11}$

$y = -\frac{3}{11}x + \frac{18}{11} - \frac{11}{11}$

$\boxed{y = -\frac{3}{11}x + \frac{7}{11}}$

3. Given the line y=4x+3, find the equation of a line parallel to this that passes through (4, 1).

$m = 4$; $(x_1, y_1) = (4, 1)$

$y - y_1 = m(x - x_1)$

$y - 1 = 4(x - 4)$

$y - 1 = 4x - 16$

$y = 4x - 16 + 1$

$y = 4x - 15$

Parallel & Perpendicular Lines:

Two lines with slopes m_1 and m_2 are **parallel** if and only if $m_1 = m_2$ (and have different y-intercepts)

*All vertical lines are parallel.

$\ell_1 \parallel \ell_2$

Two lines with slopes m_1 and m_2 are **perpendicular** if and only if $m_1 = -\frac{1}{m_2}$ or $m_2 = -\frac{1}{m_1}$ or $m_1 m_2 = -1$

*Vertical lines are perpendicular to horizontal lines

$\ell_1 \perp \ell_2$

3. Given the line y=4x+3, find the equation of a line parallel to this that passes through (4, 1).

$m = 4 \qquad (x_1, y_1) = (4, 1)$

$y - y_1 = m(x - x_1)$

$y - 1 = 4(x - 4)$

$y - 1 = 4x - 16$

$\boxed{y = 4x - 15}$

4. Given the line y=-3x+7, find the equation of a line perpendicular to it that passes through (5,-8).

$m = \frac{1}{3} \qquad (x_1, y_1) = (5, -8)$ $\qquad -\frac{8}{1} \cdot \frac{3}{3} = \frac{-24}{3}$

$y - (-8) = \frac{1}{3}(x - 5)$

$y + 8 = \frac{1}{3}x - \frac{5}{3}$

$y = \frac{1}{3}x - \frac{5}{3} - \frac{24}{3}$

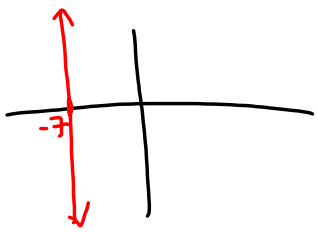
$\boxed{y = \frac{1}{3}x - \frac{29}{3}}$

Find the equation of the line:

5. Passes through (-7, 6) ; no slope

$m = \frac{y_2 - y_1}{-7-(-7)} = \frac{y_2 - y_1}{0}$

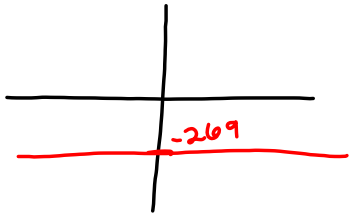
$x = -7$



6. Passes through (43, -269) ; slope 0

$m = \frac{-269-(-269)}{x_2 - x_1} = \frac{0}{x_2 - x_1} = 0$

$y = -269$



Are the two lines parallel, perpendicular, or neither?

3.6

4. $y = \frac{1}{2}$; $y = -4$

parallel

10. $y = \frac{1}{2}x + \frac{3}{2}$; $y = -\frac{1}{2}x + \frac{3}{2}$

neither

14. $4x - 3y = 2$; $4x + 3y = -7$

$-3y = -4x + 2$ $\quad$ $3y = -4x - 7$

$y = \frac{4}{3}x - \frac{2}{3}$ $\quad$ $y = -\frac{4}{3}x - \frac{7}{3}$

neither

20. $(3,5)\&(-3,3)$; $(2,-5)\&(-4,4)$

$\frac{5-3}{3-(-3)} = \frac{2}{6} = \frac{1}{3}$ $\quad$ $\frac{-5-4}{2-(-4)} = \frac{-9}{6} = -\frac{3}{2}$

neither

4.1 Solving Systems of Linear Equations
by Graphing and by the Substitution Method

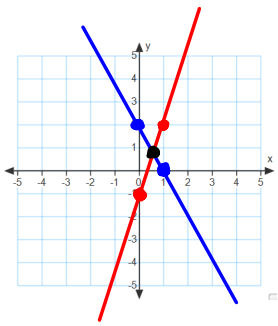
A system of equations is two or more equations considered together.

$$\begin{cases} Ax + By = C \\ Dx + Ey = F \end{cases}$$

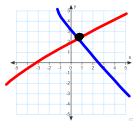
A solution of a system of equations in two variables is an ordered pair that is a solution of each equation in the system.

A solution of a system of linear equations can be found by graphing the lines of the system on the same coordinate axes. The point of intersection of the lines is the solution of the system of equations.

$$\begin{cases} y = 3x - 1 \\ y = -2x + 2 \end{cases}$$

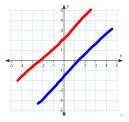


When the graphs intersect at only one point, the system of equations is called an **independent system of equations**.



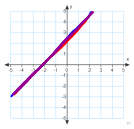
(x, y)

When the lines are parallel (and do not intersect), the system of equations is called an **inconsistent system of equations**, and has no solution.



no solution

When the two equations represent the same line, the system is called a **dependent system of equations**, and has infinitely many solutions of the form (x, mx+b).



$(x, mx+b)$

$y = mx+b$

Example 1

$$\begin{cases} y = 2x - 4 \\ y = -3x + 2 \end{cases}$$

$2x - 4 = -3x + 2$

$2x + 3x = 2 + 4$

$5x = 6$

$x = \frac{6}{5}$

$y = 2\left(\frac{6}{5}\right) - 4$

$= \frac{12}{5} - \frac{20}{5}$

$y = -\frac{8}{5}$

$\left(\frac{6}{5}, \frac{-8}{5}\right)$ (independent system)

Example 2

$\begin{cases} 2x + y = 4 \\ 3x - 2y = 5 \end{cases}$ $\Rightarrow y = -2x + 4$

$3x - 2(-2x + 4) = 5$

$3x + 4x - 8 = 5$

$7x = 13$

$x = 13/7$

$y = -2\left(\frac{13}{7}\right) + 4$

$= -\frac{26}{7} + \frac{28}{7}$

$y = \frac{2}{7}$

$\left(\frac{13}{7}, \frac{2}{7}\right)$

4.1

14. $\begin{cases} 2x + 3y = 6 \\ y = -\frac{2}{3}x + 1 \end{cases}$

$2x + 3\left(-\frac{2}{3}x + 1\right) = 6$

$2x - 2x + 3 = 6$

$3 = 6$ ↯ contradiction

inconsistent system

(parallel lines)

no solution

$\emptyset$

16. $\begin{cases} 3x - 2y = 6 \\ y = \frac{3}{2}x - 3 \end{cases}$

$3x - 2\left(\frac{3}{2}x - 3\right) = 6$

$3x - 3x + 6 = 6$

$6 = 6$

(identity)

dependent system

(same line)

infinitely many solutions ordered pairs of the form $(x, mx+b)$

$\left(x, \frac{3}{2}x - 3\right)$

4.2 Solving Systems of Equations by the Elimination Method

Rules: We can...
1. multiply an equation by a non-zero constant
2. interchange any 2 equations
3. add a non-zero multiple of any equation to another

16. $\begin{cases} 3x + 4y = 25 \\ 2x + y = 10 \end{cases} \cdot -4$

$\Rightarrow 3x + 4y = 25$

$-8x - 4y = -40$

$-5x = -15$

$x = 3$

$2(3) + y = 10$

$y = 10 - 6$

$y = 4$

$(3, 4)$

14. $\begin{cases} 3x + 6y = 7 \\ 2x + 4y = 5 \end{cases}$ $\cdot 2$ $\cdot(-3)$

$\Rightarrow 6x + 12y = 14$

$-6x - 12y = -15$

$0 = -1$ ↯

inconsistent

parallel

no solution

26. $\begin{cases} 3x + 3y = y + 1 \\ x + 3y = 9 - x \end{cases} \Rightarrow \begin{cases} (3x + 2y = 1) \cdot 2 \\ (2x + 3y = 9) \cdot (-3) \end{cases}$

$\Rightarrow 6x + 4y = 2$

$-6x - 9y = -27$

$-5y = -25$

$y = 5$

$2x + 3(5) = 9$

$2x + 15 = 9$

$2x = -6$

$x = -3$

$(-3, 5)$