Ch 5 - Exponential Expressions & Polynomials

Due Tuesday, 9/15:
5.1 #63-85 odd

5.2 #3-7odd, 15-25odd, 35-49odd — due tomorrow (Wed.)
5.3 #25-29odd, 43-51odd, 61-67odd, 89-97odd, 109-117odd
5.4 #19-25 odd; 27-43 odd; 55-61 odd
5.5 #21-47 odd

Product	$b^p \cdot b^q = b^{p+q}$
Quotient	$\frac{b^p}{b^q} = b^{p-q} = \frac{1}{b^{q-p}}$
Power	$(b^p)^q = b^{pq}$
Distributive	$(a^p b^q)^r = a^{pr} b^{qr}$, $\left(\frac{a^p}{b^q}\right)^r = \frac{a^{pr}}{b^{qr}}$
Reciprocal	$b^{-p} = \frac{1}{b^p}$

$\frac{1}{b^{-p}} = b^p$

$$\frac{y^{2n}}{-y^{8n}} = -\frac{y^{2n}}{y^{8n}} = -\frac{y^{2n-8n}}{1} = -\frac{1}{y^{8n-2n}}$$

$$= -y^{-6n} = \boxed{\frac{-1}{y^{6n}}}$$

$$\frac{x^{2n-1}y^{n-3}}{x^{n+4}y^{n+3}} = \frac{x^{2n-1-(n+4)}}{y^{n+3-(n-3)}} = \frac{x^{n-5}}{y^{6}}$$

$$81\left(\frac{9ab^{-2}}{8a^{-2}b}\right)^{-2}\left(\frac{3a^{-2}b}{2a^{2}b^{-2}}\right)^{3}$$

$$=\frac{9^{-2}a^{-2}b^{4}}{8^{-2}a^{4}b^{-2}}\cdot\frac{3^{3}a^{-6}b^{3}}{2^{3}a^{6}b^{-6}}=$$

$$=\frac{8^{2}b^{6}}{9^{2}a^{6}}\cdot\frac{3^{3}b^{9}}{2^{3}a^{12}}=\frac{(2^{3})^{2}\cdot3^{3}b^{15}}{(3^{2})^{2}\cdot2^{3}a^{18}}=\frac{2^{6}\cdot3^{3}b^{15}}{3^{4}\cdot2^{3}a^{18}}$$

$$=\frac{2^{3}b^{15}}{3^{1}a^{18}}=\boxed{\frac{8b^{15}}{3a^{18}}}$$

5.2 Introduction to Polynomials

A polynomial is an expression consisting of variables and constants, using only the operations of addition, subtraction, multiplication, and non-negative integer exponents.

A polynomial with one term is a monomial.

e.g. $3x^2$ or $5xy^3$

A polynomial with two terms is a binomial.

e.g. $7xy^5 - 3x$ or $xyzw + 23x^2$ or $x - 2$

A polynomial with three terms is a trinomial.

e.g. $x^2 + 5x - 6$

The <u>degree of a monomial</u> is the sum of the exponents of the variables.

$7x^{1}y^5$ has degree 6

$x^{1}y^{1}z^{1}w^{1}$ has degree 4

$13x^3y^{1}z^2$ has degree 6

$-2a^{1}b^3$ has degree 4

The <u>degree of a polynomial</u> is the greatest of the degrees of any of its terms.

$x^2 + 5x - 6$ has degree 2

$\overset{2}{3xy} - \overset{4}{15x^3y} + \overset{5}{2v^3xz}$ has degree 5

$\overset{3}{15xy^2} - \overset{1}{\sqrt{2}x} + \overset{3}{32xyz} - \overset{0}{5000}$ has degree 3

The terms of a polynomial in only one variable are usually arranged in <u>descending order</u>, so that the exponents of the variable decrease from left to right, in the form

$$f(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_2x^2 + a_1x + a_0$$

$a_n, \ldots, a_0$ are real-numbered <u>coefficients</u>

a_nx^n is the <u>lead term</u> (term containing the variable with the largest exponent)

a_n is the <u>leading coefficient</u> (coefficient of the variable with the largest exponent)

a_0 is the <u>constant term</u> (term without a variable)

n is the <u>degree</u> of the polynomial (largest exponent)

The <u>linear function</u> $f(x) = mx + b$ is a polynomial of degree one.
A second-degree polynomial of the form $f(x) = ax^2 + bx + c$ is called a <u>quadratic function</u>.
A third-degree polynomial is called a <u>cubic function</u>.

Problems from Section 5.2:

Is it a polynomial? If so, state the lead term, leading coefficient, degree, and constant term.

16. $P(x) = 3x^4 - 3x - 7$

Lead term: $3x^4$
Leading coefficient: 3
Degree: 4
Constant term: -7

18. $R(x) = \frac{3x^2-2x+1}{x}$

Lead term:
Leading coefficient:
Degree:
Constant term:

Not a polynomial!

20. $f(x) = x^2 - \sqrt{x+2} - 8$

Lead term:
Leading coefficient:
Degree:
Constant term:

not a polynomial!

22. $g(x) = -4x^5 + 3x^2 + x - \sqrt{7}$

Lead term: $-4x^5$
Leading coefficient: -4
Degree: 5
Constant term: $-\sqrt{7}$

To evaluate a polynomial, replace the variable by its value and simplify.

6. Given $R(x) = -x^3 + 2x^2 - 3x + 4$, evaluate $R(-1)$.

$R(-1) = -(-1)^3 + 2(-1)^2 - 3(-1) + 4 =$

$= (-1)(-1) + 2(1) + (-1)(3)(-1) + 4$

$= 1 + 2 + 3 + 4$

$= \boxed{10}$

Polynomials can be added by combining like terms.

36. $(3x^2 - 2x + 7) + (-3x^2 + 2x - 12)$

$0 + 0 - 5 = \boxed{-5}$

42. $(3a^2 - 9a) - (-5a^2 + 7a - 6) = 3a^2 - 9a + 5a^2 - 7a + 6$

$= \boxed{8a^2 - 16a + 6}$

50. $(2x^4 - 2x^2 + 1) - (3x^3 - 2x^2 + 3x + 8) = 2x^4 - 2x^2 + 1 - 3x^3 + 2x^2 - 3x - 8$

$= \boxed{2x^4 - 3x^3 - 3x - 7}$

46. $(2x^{2n} - x^n - 1) - (5x^{2n} + 7x^n + 1)$

$= \boxed{-3x^{2n} - 8x^n - 2}$

Review

Simplify: $\frac{2x^{-5}y^4}{3x^2y} \cdot \frac{3^{-2}xy^3}{2^3x} = \frac{y^{4+3-1}}{2^{3-1} \cdot 3^{1-(-2)} \cdot x^{2+1-(-5)-1}}$

$= \frac{y^6}{2^2 3^3 x^7} = \frac{y^6}{4 \cdot 27x^7} = \boxed{\frac{y^6}{108x^7}}$

Find the equation of the line passing through the points $(5, -3)$ & $(-1, 1)$.

$\frac{-3-1}{5-(-1)} = \frac{-4}{6} = \frac{-2}{3}$

$y - 1 = -\frac{2}{3}(x - (-1))$

$y - 1 = -\frac{2}{3}x - \frac{2}{3}$

$y = -\frac{2}{3}x - \frac{2}{3} + 1$

$\boxed{y = -\frac{2}{3}x + \frac{1}{3}}$

5.3 Multiplication of Polynomials

Distributive Property: $a(b+c) = ab + ac$

$a(b+c+d+e+f) = ab + ac + ad + ae + af$

Multiplying a Polynomial by a Monomial

$-3xy^2(2x^3y - xy^4 + 4x^3y^2)$

$= (-3xy^2)(2x^3y) + (-3xy^2)(-xy^4) + (-3xy^2)(4x^3y^2)$

$= \boxed{-6x^4y^3 + 3x^2y^6 - 12x^4y^4}$

In general, multiply every term by every other term and then combine like terms.

$(3x^5 - 2x^3 + 3)(4x^2 - 5x)$

$(3x^5 - 2x^3 + 3)(4x^2) + (3x^5 - 2x^3 + 3)(-5x)$

$(3x^5)(4x^2) + (-2x^3)(4x^2) + 3(4x^2) + (3x^5)(-5x) + (-2x^3)(-5x) + 3(-5x)$

$= 12x^7 - 8x^5 + 12x^2 - 15x^6 + 10x^4 - 15x$

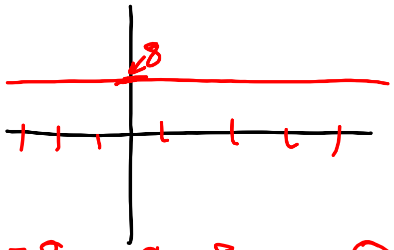
$= 12x^7 - 15x^6 - 8x^5 + 10x^4 + 12x^2 - 15x$

$(2x - 3 + 4x^2)(5x^3 - x^8 + 2x)$

$2x(5x^3) + 2x(-x^8) + 2x(2x) + (-3)(5x^3) + (-3)(-x^8) + (-3)(2x) + (4x^2)(5x^3) + (4x^2)(-x^8) + (4x^2)(2x)$

$= 10x^4 - 2x^9 + 4x^2 - 15x^3 + 3x^8 - 6x + 20x^5 - 4x^{10} + 8x^3$

$= -4x^{10} - 2x^9 + 3x^8 + 20x^5 + 10x^4 - 7x^3 + 4x^2 - 6x$

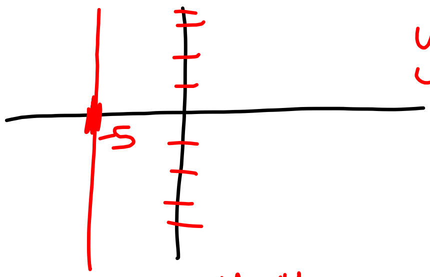


$y - y_1 = m(x - x_1)$

0

$y - y_1 = 0$ 0

$y = 8$ $\frac{8-8}{x_2 - x_1} = \frac{0}{x_2 - x_1} = 0$ $y = y_1$



$\cancel{y - y_1 = m(x - x_1)}$

$x = -5$ $\frac{y_2 - y_1}{-5 - (-5)} = \frac{y_2 - y_1}{0}$ = undefined (no slope)