HW #10 – due Fri, 10/2
5.7 #35-75odd Solving equations by factoring

HW #11 – due Mon, 10/5
6.1 #39-79 odd Simplifying rational expressions

HW #12 – due Tues, 10/6
6.2 #3-95 odd Operations on rational expressions

6.1, 6.2 Rational Functions

A rational function is of the form $f(x) = \frac{p(x)}{q(x)}$, where p & q are polynomials.

Simplify. Factor First!

$\frac{8}{12} = \frac{2 \cdot 4}{3 \cdot 4} = \frac{2}{3}$

46. $\frac{3a^2 - 6a}{12 - 6a}$

$= \frac{3a(a-2)}{-6(-2+a)} = \frac{3a(a-2)}{-2 \cdot 3(a-2)}$

$= \frac{a}{-2} = \frac{-a}{2} = \boxed{-\frac{a}{2}}, \quad a \neq 2$

52. $\frac{3x^3y^3 - 12x^2y^2 + 15xy}{3xy}$

$= \frac{3xy(x^2y^2 - 4xy + 5)}{3xy}$

$= \boxed{x^2y^2 - 4xy + 5}, \quad x \neq 0, \quad y \neq 0$

60. $\dfrac{2x^2+7xy-4y^2}{4x^2-4xy+y^2}$

$= \dfrac{2x^2+8xy-1xy-4y^2}{4x^2-2xy-2xy+y^2}$

$= \dfrac{2x(x+4y)-y(x+4y)}{2x(2x-y)-y(2x-y)}$

$= \dfrac{(x+4y)(2x-y)}{(2x-y)(2x-y)} = \boxed{\dfrac{x+4y}{2x-y}}$, $y \neq 2x$

$2x-y \neq 0$

$2x \neq y$

$x \neq y/2$

72. $\dfrac{4a^2-8ab+4b^2}{4a^2-4b^2}$

$= \dfrac{4(a^2-2ab+b^2)}{4(a^2-b^2)}$

$= \dfrac{(a-b)(a-b)}{(a+b)(a-b)} = \boxed{\dfrac{a-b}{a+b}}$, $a \neq b, -b$

$a+b \neq 0$

$a-b \neq 0$

78. $\dfrac{x^4-2x^2-3}{x^4+2x^2+1}$

$= \dfrac{(x^2+1)(x^2-3)}{(x^2+1)(x^2+1)} = \dfrac{x^2-3}{x^2+1} = \dfrac{(x+\sqrt{3})(x-\sqrt{3})}{(x+i)(x-i)}$

nothing to exclude

domain is $(-\infty, \infty)$

6.2 Operations on Rational Functions

(Same rules of fractions apply)

$$\frac{a}{b} \pm \frac{c}{d} = \frac{a}{b}\cdot\frac{d}{d} \pm \frac{c}{d}\cdot\frac{b}{b} = \frac{ad}{bd} \pm \frac{cb}{db} = \frac{ad \pm cb}{db}$$

$$\frac{a}{b}\cdot\frac{c}{d} = \frac{ac}{bd} = \frac{a}{1}\cdot\frac{c}{bd} = \frac{a}{bd}\cdot\frac{c}{1} = \frac{ac}{1}\cdot\frac{1}{bd} = \frac{a}{d}\cdot\frac{c}{b}$$

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b}\cdot\frac{d}{c} = \frac{ad}{bc}$$

8. $\dfrac{2x^2-5x+3}{x^6y^3} \cdot \dfrac{x^4y^4}{2x^2-x-3}$

$\dfrac{(2x^2-2x-3x+3)}{(2x^2+2x-3x-3)} \cdot \dfrac{x^4y^4}{x^6y^3} \cdot y^1$

$\dfrac{2x(x-1)-3(x-1)}{2x(x+1)-3(x+1)} \cdot \dfrac{y}{x^2}$

$\dfrac{(x-1)(2x-3)}{(x+1)(2x-3)} \cdot \dfrac{y}{x^2} = \boxed{\dfrac{y(x-1)}{x^2(x+1)}} = \boxed{\dfrac{xy-y}{x^3+x^2}}$

$x \neq 0, -1, 3/2$

$y \neq 0$

18. $\dfrac{x^4-5x^2+4}{3x^2-4x-4} \cdot \dfrac{3x^2-10x-8}{x^2-4}$

$\dfrac{(x-2)(x+2)(x-1)(x+1)}{(x^2-4)(x^2-1)}$ $\quad \dfrac{(x-2)(3x-4)}{3x(x-2)-4(x-2)}$ $\quad 3x^2-6x-4x-8$

$3x^2-6x+2x-4$ $\quad 3x(x-2)+2(x-2)$ $\quad (x-2)(3x+2)$

$(x-2)(x+2)$

$x-2 \neq 0$ $\quad 3x+2 \neq 0$ $\quad x+2 \neq 0$

$= \dfrac{(x-2)(x+2)(x-1)(x+1)(x-2)(3x-4)}{(x-2)(3x+2)(x-2)(x+2)}$

$= \boxed{\dfrac{(x-1)(x+1)(3x-4)}{3x+2}}$, $x \neq 2, -2, -2/3$