HW #10 - due Fri, 10/2
5.7 #35-75odd Solving equations by factoring

HW #11 - due Mon, 10/5
6.1 #39-79 odd Simplifying rational expressions

HW #12 - due Fri, 10/8
6.2 #3-95 odd Operations on rational expressions
6.6 #5-25 odd Literal Equations

7.1 #85-113 odd Rational Exponents and Radical Expressions
39-73 odd, 125-149 odd
7.2 #11-21 odd, 43-51 odd, 57-65 odd, 85-91 odd, 97-103 odd,113-121 odd
8.2 #59-69 odd

7.2

16. $\sqrt{60xy^7z^{12}} = \sqrt{2^2 \cdot 15x(y^3)^2 \cdot y \cdot (z^6)^2}$

$= |2y^3z^6|\sqrt{15xy}$

$= 2|y^3|z^6\sqrt{15xy}$

20. $\sqrt[3]{a^8 b^{11} c^{15}} = \sqrt[3]{(a^2)^3 a^2 (b^3)^3 b^2 (c^5)^3}$

$\sqrt[3]{a^6 a^2 b^9 b^2 c^{15}}$

$= \boxed{a^2 b^3 c^5 \sqrt[3]{a^2 b^2}}$

22. $\sqrt[4]{64x^8 y^{10}}$

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

4, 8, 16, 32, 64

$= \sqrt[4]{2^6 x^8 y^{10}}$

$= \sqrt[4]{2^4 \cdot 2^2 (x^2)^4 (y^2)^4 y^2}$

$= |2x^2y^2| \sqrt[4]{4y^2}$

$= \boxed{2x^2y^2 \sqrt[4]{4y^2}}$

7.2 Operations on Radical Expressions

Properties of Radicals:

Let $m, n \in \mathbb{N}$ *and* $a, b \in \mathbb{R}$ *such that* $a^{\frac{1}{n}}, b^{\frac{1}{n}} \in \mathbb{R}$.

$\sqrt[n]{b} = b^{\frac{1}{n}}$

$\left(\sqrt[n]{b}\right)^m = \sqrt[n]{b^m} = b^{\frac{m}{n}}$

$\sqrt[n]{b^n} = \begin{cases} b, & if\ n\ is\ odd \\ |b|, & if\ n\ is\ even \end{cases}$

$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

$$a^{1/n} b^{1/n} = (ab)^{1/n}$$

$$\frac{a^{1/n}}{b^{1/n}} = \left(\frac{a}{b}\right)^{1/n}$$

$$a^{1/mn} = \left(a^{1/n}\right)^{1/m} = \sqrt[m]{\sqrt[n]{a}}$$

Add & Subtract Radicals by Combining Like Terms

7.2

26. $3\sqrt[3]{11} - 8\sqrt[3]{11}$ $= 3\left(\sqrt[3]{11}\right) - 8\left(\sqrt[3]{11}\right)$

$= \left(\sqrt[3]{11}\right)(3-8)$

$3x - 8x$

$= \boxed{-5\sqrt[3]{11}}$

32. $\sqrt{48x} + \sqrt{147x}$

$16 \cdot 3 \qquad 49 \cdot 3$

$= \sqrt{4^2 \cdot 3x} + \sqrt{7^2 \cdot 3x}$

$= 4\sqrt{3x} + 7\sqrt{3x}$

$= \boxed{11\sqrt{3x}}$

34. $\sqrt{18b} + \sqrt{75b}$

$= \sqrt{3^2 \cdot 2b} + \sqrt{5^2 \cdot 3b}$

$= \boxed{3\sqrt{2b} + 5\sqrt{3b}}$

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \quad 2$

4

8

16

2^4 32

64

128

2^7

$2^6 \cdot 2$

46. $2b\sqrt[3]{16b^2} + \sqrt[3]{128b^5}$

$= 2b\sqrt[3]{2^3 \cdot 2b^2} + \sqrt[3]{(2^2)^3 \cdot 2 \cdot b^3 b^2}$

$= 4b\sqrt[3]{2b^2} + 4b\sqrt[3]{2b^2}$

$= \boxed{8b\sqrt[3]{2b^2}}$

62. $\sqrt{a^3b}\sqrt{ab^4}$

$= \sqrt{(a^3b)(ab^4)}$

$= \sqrt{a^4b^5}$

$= \sqrt{(a^2)^2(b^2)^2 b} = \boxed{a^2b^2\sqrt{b}}$

64. $\sqrt{5x^3y}\sqrt{10x^3y^4}$

$\sqrt[n]{x^n} = \begin{cases} x, & n \text{ odd} \\ |x|, & n \text{ even} \end{cases}$

$= \sqrt{(5x^3y)(10x^3y^4)}$

$= \sqrt{50x^6y^5} = \sqrt{5^2 \cdot 2(x^3)^2 \cdot (y^2)^2 \cdot y}$

$= 5|x^3|y^2\sqrt{2y}$

74. $\sqrt{3a}\left(\sqrt{27a^2}-\sqrt{a}\right)$; $a>0$

$= \sqrt{3a}\sqrt{27a^2} - \sqrt{3a}\sqrt{a}$

$= \sqrt{(3a)(27a^2)} - \sqrt{(3a)(a)}$

$= \sqrt{81a^3} - \sqrt{3a^2}$

$= \sqrt{9^2a^2a} - \sqrt{3a^2}$

$= \boxed{9a\sqrt{a} - a\sqrt{3}}$

82. $\left(\sqrt{2}-3\right)\left(\sqrt{2}+4\right)$

$\underbrace{\sqrt{2}\sqrt{2}}_{\sqrt{2^2}} + 4\sqrt{2} - 3\sqrt{2} + (-3)(4)$

$2 + 1\sqrt{2} - 12$

$\boxed{\sqrt{2} - 10}$

Rationalize the Denominator

assume variables are positive

(rewrite so that there are no radicals in the denominator)

100. $\frac{2}{\sqrt{3y}} \cdot \frac{\sqrt{3y}}{\sqrt{3y}} = \frac{2\sqrt{3y}}{\sqrt{(3y)^2}} = \boxed{\frac{2\sqrt{3y}}{3y}}$

102. $\frac{9}{\sqrt{3a}} \cdot \frac{\sqrt{3a}}{\sqrt{3a}} = \frac{9\sqrt{3a}}{3a} = \boxed{\frac{3\sqrt{3a}}{a}}$

When the *entire* denominator is a radical, multiply by $1 = \frac{\sqrt{\square}}{\sqrt{\square}}$ $(a-b)(a+b) = a^2 - b^2$

When the denominator contains radicals added or subtracted with something, we multiply by the conjugate of the denominator over itself. $a + b$ and $a - b$ are conjugates.

114. $\cancel{\frac{5}{(2-\sqrt{7})} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{5\sqrt{7}}{2\sqrt{7} - 7}}$

$\frac{5}{(2-\sqrt{7})} \cdot \frac{(2+\sqrt{7})}{(2+\sqrt{7})} = \frac{10+5\sqrt{7}}{2^2 - (\sqrt{7})^2} = \frac{10+5\sqrt{7}}{4-7}$

$= \boxed{\frac{10+5\sqrt{7}}{-3}} = -\frac{10+5\sqrt{7}}{3} = \frac{-10-5\sqrt{7}}{3}$

120. $\frac{(3-\sqrt{x})}{(3+\sqrt{x})} \cdot \frac{(3-\sqrt{x})}{(3-\sqrt{x})}$

$(a-b)(a-b) = (a-b)^2 = a^2 - 2ab + b^2$

$(a-b)(a+b) = a^2 - b^2$

$$= \frac{3^2 - 2(3)\sqrt{x} + (\sqrt{x})^2}{3^2 - (\sqrt{x})^2}$$

$$= \boxed{\frac{9 - 6\sqrt{x} + x}{9 - x}}$$

122. $\frac{\sqrt{2}+\sqrt{3}}{\sqrt{3}-\sqrt{2}} \cdot \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)(a+b) = a^2 - b^2$

$$= \frac{(\sqrt{2})^2 + 2\sqrt{2}\sqrt{3} + (\sqrt{3})^2}{(\sqrt{3})^2 - (\sqrt{2})^2}$$

$$= \frac{2 + 2\sqrt{6} + 3}{3 - 2} = \boxed{5 + 2\sqrt{6}}$$

$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab} = (ab)^{1/n} = a^{1/n} b^{1/n}$

8.2 Solving Quadratic Equations using the Quadratic Formula

Given a quadratic equation in standard form,

$$ax^2 + bx + c = 0, \ \ a \neq 0$$

Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

60. $(1)z^2 - 4z - 8 = 0$

$a=1 \quad b=-4 \quad c=-8$

$z = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-8)}}{2(1)}$

$= \frac{4 \pm \sqrt{16+32}}{2} = \frac{4 \pm \sqrt{48}}{2}$

$= \frac{4 \pm 4\sqrt{3}}{2} = \frac{\cancel{2}(2 \pm 2\sqrt{3})}{\cancel{2}} = \boxed{2 \pm 2\sqrt{3}}$

$2+2\sqrt{3}$ & $2-2\sqrt{3}$

68. $4p^2 - 7p = -3$

$4 \cdot p^2 - 7p + 3 = 0$

$a=4, \ b=-7, \ c=3$

$p = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(4)(3)}}{2(4)}$

$= \frac{7 \pm \sqrt{49-48}}{8} = \frac{7 \pm 1}{8}$

$= \frac{7+1}{8}, \frac{7-1}{8} = \frac{8}{8}, \frac{6}{8} = \boxed{1, \frac{3}{4}}$